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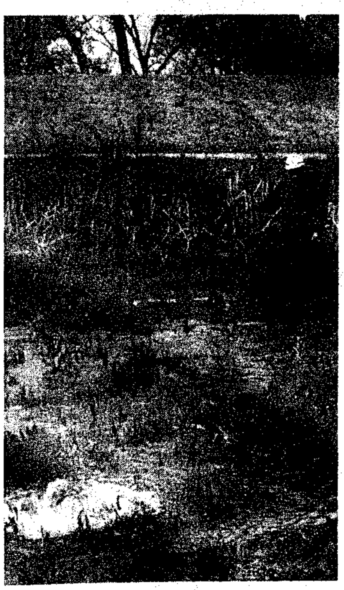

REMR RESEARCH PROGRAM

REPAIR, EVALUATION, MAINTENANCE, AND REHABILITATION RESEARCH PROGRAM

TECHNICAL REPORT REMR-GT-1

MATHEMATICAL ANALYSES OF LANDSIDE SEEPAGE BERMS

by

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The following two letters used as part of the number designating technical reports of research published under the Repair, Evaluation, Maintenance, and Rehabilitation (REMR) Research Program identify the problem area under which the report was prepared:

	Problem Area		Problem Area
CS	Concrete and Steel Structures	EM	Electrical and Mechanical
GT	Geotechnical	EI	Environmental Impacts
HY	Hydraulics	OM	Operations Management
CO	Coastal		

For example, Technical Report REMR-CS-1 is the first report published under the Concrete and Steel Structures problem area.

COVER PHOTOS:

TOP — Levee protection efforts during high water conditions, Mississippi River, Rock island District.

BOTTOM — Adverse underseepage along a mainline levee manifested in an uncontrolled sand boil.

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Levee underseepage
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Seepage control

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes a study that extends solutions for mathematical analyses of seepage berms presented in "Investigation of Underseepage and Its Control, Lower Mississippi River Levees," Technical Memorandum 3-424, Vol 1, October 1956, US Army Engineer Waterways Experiment Station.

A plot of the required seepage berm width, B , versus the ratio of the permeability of the berm to the top blanket, $\bar{K}$, for various safety factors indicates B is very sensitive to $\bar{K}$ for $\bar{K} \leq 1$. When the (Continued)

20. ABSTRACT (Continued)

uplift safety factor varies from 1.5 at the landside levee toe to 1.0 at the landside seepage berm toe, the berm width is only slightly greater than that for a uniform safety factor of 1.0. If the uplift safety factor is greater than 1.0 at the berm toe, then as the top blanket becomes thinner, the berm width becomes longer. Thus, other methods of underseepage control should be investigated to determine whether they may be more economical.

When the seepage berm is impervious, the berm width is a maximum. When the seepage berm is infinitely pervious, the berm width is a minimum. Therefore, seepage berms should be constructed of the most pervious soils available (with adequate provisions for control of surface erosion and internal piping) in the interest of economy.

Because of the great difficulty in determining the permeability of the foundation, top blanket, and seepage berm, the mathematical solutions presented in this report should be used only as a guide to good engineering judgement. A range of permeability values should be used rather than average values.

Supplements 1, 2, and 3 have been added to this corrected version to present solutions for seepage berms with constant slope of upper surface, riverside seepage berms, and general cases and short berms, respectively.

Preface

The first draft of this report was prepared by Mr. Reginald A. Barron (now deceased), Consulting Engineer, as part of the work being performed by the US Army Engineer Waterways Experiment Station (WES) on revision of Engineer Manual 1110-2-1901, "Seepage Control." Funds for this work were provided by Headquarters, US Army Corps of Engineers (HQUSACE), under CWIS Work Unit 31836. Mr. Barron was subsequently funded by WES via Purchase Orders DACW39-79-M-4486, dated 26 July 1979, DACW39-81-M-1323, and DACW39-81-M-3076. The report was completed and published at WES under the auspices of the Repair, Evaluation, Maintenance, and Rehabilitation (REMR) Research Program.

This report supersedes WES Miscellaneous Paper GL-80-15. The main text of that report has been corrected and three supplements have been added to produce this version.

The contracts under which the first draft was prepared were managed by Dr. Edward B. Perry, Soil Mechanics Division (SMD), Geotechnical Laboratory (GL), under the general supervision of Mr. Clifford L. McAnear, Chief, SMD, and Dr. William F. Marcuson III, Chief, GL.

Special acknowledgement is made to the earlier work done on the seepage berm theory by Messrs. P. T. Bennett (retired), Missouri River Division, and R. I. Kaufman, Lower Mississippi Valley Division (LMVD). Valuable assistance was given by Mr. C. K. Smith (retired), HQUSACE, who reviewed the first draft of the report and suggested that the boundary conditions at the seepage berm toe and levee toe be considered in a more exact manner for the finite difference solutions than the approximate manner in the first draft. Technical assistance was provided by the geotechnical staffs of Rock Island District, North Central Division, and LMVD.

Commanders and Directors of WES during the preparation and publication of this report were COL Nelson P. Conover, CE, COL Tilford C. Creel, CE, and COL Robert C. Lee, CE. Mr. F. R. Brown was Technical Director.

Contents

Conversion Factors, Non-SI to SI (Metric) Units of Measurement

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

Multiply	By	To Obtain
cubic yards	0.7645549	cubic metres
feet	0.3048	metres
pounds (mass) per cubic foot	16.01846	kilograms per cubic metre
square feet	0.09290304	square metres

MATHEMATICAL ANALYSES OF LANDSIDE SEEPAGE BERMS

Introduction

1. This report is a compilation of studies done by the author in the early 1950's and late 1970's. In the 1950's, work was also done by Mr. P. T. Bennett, Missouri River Division, and by Mr. R. I. Kaufman, US Army Engineer Waterways Experiment Station, now at the Lower Mississippi Valley Division. The results of these early studies are presented in TM 3-424.* These early studies were mainly concerned with seepage berms that had a coefficient of permeability equal to that of the landside top blanket. Studies by the author in 1979 have extended the solutions to include cases where the berm permeability is not that of the top blanket. In one case, the solution of the differential equation has not been obtained and an approximation has been developed using finite differences. Supplements No. 1, 2, and 3 to this report present solutions for seepage berms with constant slope of upper surface, riverside seepage berms, and general cases and short berms, respectively.

Assumptions

2. The foundation conditions for dams and levees are so complex that it is necessary to make simplifying assumptions so that mathematical solutions may be obtained to determine the influence of downstream seepage berms on seepage and seepage uplift heads. Because of these assumptions, any solution obtained is an approximation of the real conditions. The solutions should be regarded as aids to engineering judgement. The assumptions are:

- a. Two-dimensional seepage is in a vertical plane.
- b. The top blanket and the pervious foundation extend in a landward direction to infinity.

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* US Army Engineer Waterways Experiment Station. 1956. "Investigation of Underseepage and Its Control, Lower Mississippi River Levees," Technical Memorandum 3-424, Vol 1, Vicksburg, MS.

c. The top blanket is at least ten times less pervious than the lower pervious foundation.

d. The top blanket is pervious only in a vertical direction. Thus, the permeability in the horizontal direction is zero.

e. Except where specifically noted, the permeability of the seepage berm conforms to d above. The permeability of the berm may be different from that of the top blanket.

f. The pervious foundation is pervious only in the vertical direction, and the permeability in the horizontal direction is zero.

g. The pervious foundation rests upon an impervious foundation.

h. The seepage berm, the semipervious top blanket, and the pervious foundation are homogeneous.

i. The central part of the levee and the underlying top blanket are impervious.

j. The landside water table is at, or above, the top of the top blanket.

k. Except where specifically stated otherwise, the seepage flows upward through the seepage berm to emerge on its upper surface. Thus, the upper surface of the seepage berm is not an equipotential surface.

l. The seepage through the foundation, top blanket, and seepage berm is not time-dependent.

Case I - Impervious Berm

3. The permeability of the seepage berm is zero for this case. However, there may be cases where the berm thickness is so great, and because it is assumed that the berm horizontal permeability is zero, that there will be no upward seepage in the berm. The latter case will have the same foundation seepage and uplift at the base of the top blanket as that for the case where the berm is impervious. The uplift head at the downstream (or landward) toe of the seepage berm is h_a. The seepage uplift safety factor, F, at this location is expressed as

$$F = \frac{Z_b \gamma_b'}{h_a \gamma_w} \qquad (1)$$

where

Z_b = thickness of the semipervious top blanket

γ_b' = buoyant weight of the semipervious top blanket

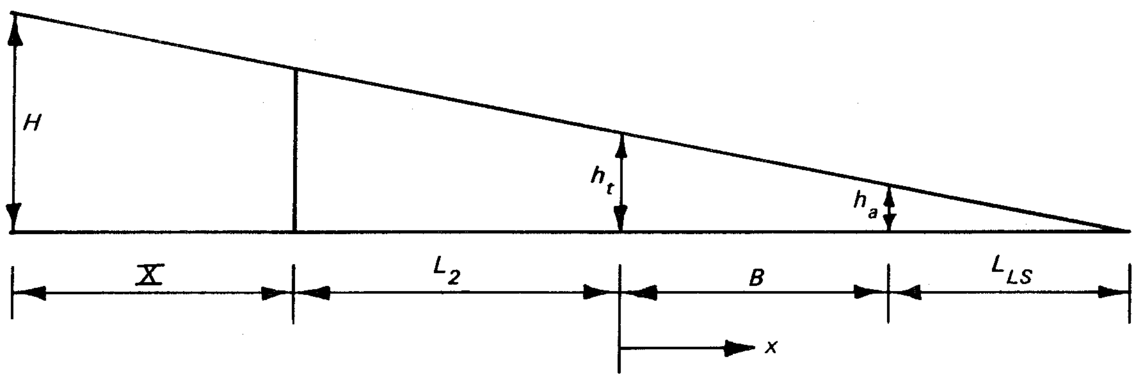
h_a = allowable seepage head at the landside seepage berm toe

γ_w = unit weight of water

Rearranging Equation 1 results in

$$h_a = \frac{Z_b \gamma_b'}{\gamma_w F} \tag{2}$$

4. A schematic hydraulic potential profile is shown below (no upward seepage under or through the seepage berm).



where

H = net hydraulic head

h_t = uplift under the semipervious top blanket at the landward or downstream toe of the impervious levee or dam

h_a = uplift under the semipervious top blanket at the landward or downstream toe of the seepage berm

B = downstream width of the seepage berm

L_2 = base width of the impervious levee or dam

L_{LS} = effective landside length of the semipervious top blanket

$\underline{\overline{X}}$ = effective riverside or upstream length of the semipervious top blanket

x = horizontal distance measured landward from downstream toe of the levee or dam

H and h_a are related as

$$\frac{h_a}{L_{LS}} = \frac{H}{\underline{\overline{X}} + L_2 + B + L_{LS}} \tag{3}$$

Thus, the seepage berm length, B, becomes

$$B = \frac{HL_{LS}}{h_a} - (\underline{\overline{X}} + L_2 + L_{LS}) \tag{4}$$

Substituting the value of h_a from Equation 2 into Equation 4 results in

$$B = \frac{HL_{LS}\gamma_w F}{Z_b \gamma'_b} - (\underline{\overline{X}} + L_2 + L_{LS}) \tag{5}$$

The seepage uplift under the seepage berm varies in a linear manner from h_t at $x = 0$ at the landward levee toe to h_a at $x = B$ at the landward toe of the seepage berm. This condition is a result of the berm being impervious with no upward seepage through the top blanket under the berm. The uplift, h_x at point x, where $0 < x < B$, is expressed as

$$h_x = h_t - (h_t - h_a)\frac{x}{B} \tag{6}$$

5. The permeability of the seepage berm is assumed to be zero (or nearly so), but sufficient seepage upward is assumed to render it buoyant. The uplift safety factor for the seepage berm and the semi-pervious top blanket at $0 < x < B$ is

$$F_x = \frac{t_x \gamma'_t}{(h_x - t_x)\gamma_w} \tag{7}$$

The berm thickness, t_x at point x, is written as

$$t_x = \frac{h_x \gamma_w F_x}{\gamma_t' + \gamma_w F_x} \qquad (8)$$

where F_x is the safety factor at point x that may be a constant or a variable with x .

Example: (to be used throughout this report)

$H = 30$ ft*, $Z_b = 5$ ft, $D = 50$ ft, $k_f/k_b = 200$,

$L_2 = 200$ ft, $\gamma_w = \gamma_b' = \gamma_t'$, and $L_1 = 500$ ft

where

D = thickness of the pervious foundation

k_f = horizontal permeability coefficient of the pervious foundation

k_b = vertical permeability coefficient of the top blanket

γ_t' = buoyant unit weight of the seepage berm

L_1 = length of the riverside top blanket

The length of the landside top blanket is infinite. A generalized cross section of the geologic strata, levee, and seepage berm is shown in Figure 1.

6. The effective riverside length of the semipervious top blanket is expressed as

$$\underline{\overline{X}} = \frac{\tanh (cL_1)}{c}$$

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* A table of factors for converting non-SI units of measurement to SI (metric) units is presented on page 3.

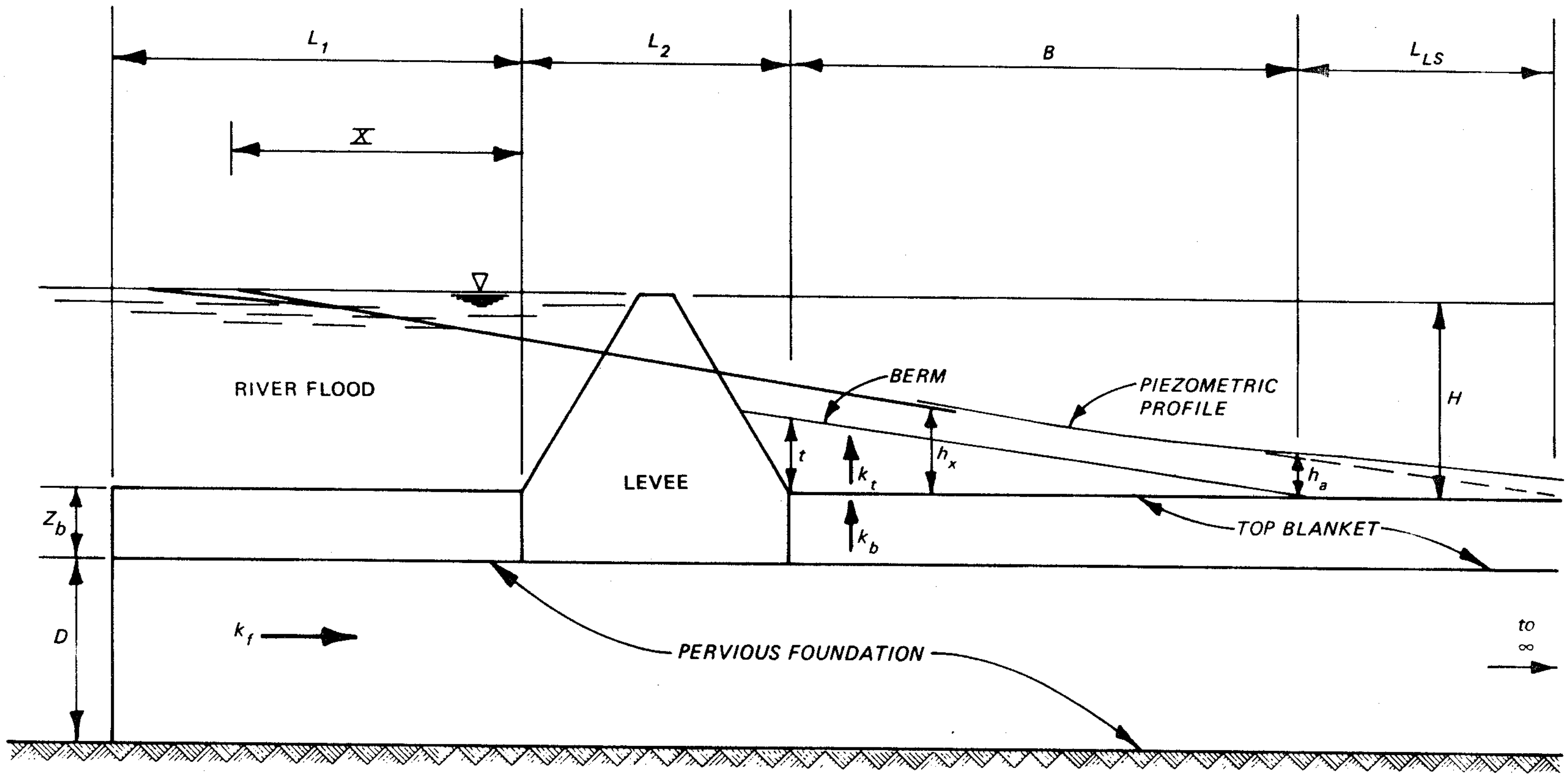


Figure 1. Generalized cross section of geologic strata, levee, and seepage berm

where

$$c = \left(\frac{k_b}{k_f D Z_b}\right)^{1/2}$$

$$= \frac{1}{\sqrt{200 \times 50 \times 5}}$$

$$= \frac{1}{223.6 \text{ ft}}$$

$$\underline{\overline{X}} = 223.6 \tanh (500 \div 223.6) = 218.5 \text{ ft}$$

The effective landside length of the semipervious top blanket is then

$$L_{LS} = \frac{1}{c} = 223.6 \text{ ft}$$

By substituting the values above in Equation 5, the width of the seepage berm becomes

$$B = \frac{30 \times 223.6 \times 1 \times F}{5 \times 1} - (218.5 + 200 + 223.6)$$

$$= 1341.6 \text{ F} - 642.1$$

For a safety factor that is uniform for $0 < x < B$, the required berm length, B , is given in the following tabulation. It should be noted that the value of B is a maximum for a given safety factor because of the imperviousness of the berm. Also, B will be less for other cases where the berm permeability is greater than zero.

7. The uplift at the landside levee toe $(x = 0)$ is expressed as

$$h_t = \frac{H (B + L_{LS})}{\underline{\overline{X}} + L_2 + B + L_{LS}} = \frac{30 (B + 223.6)}{B + 642.1}$$

8. For a constant safety factor, the berm thickness varies in a

linear manner from a maximum value at the landside toe of the levee to a minimum at $x = B$. The berm thickness at $x = 0$ is t; by using Equation 8, the values of t are obtained as given in the following tabulation.

Safety Factor F	Berm Length B, ft	Uplift at $x = 0$ h_t, ft	Berm Thickness t, ft	Berm Thickness $t_x = B$, ft
1.0	700	20.65	10.3	2.5
1.1	834	21.49	11.3	2.4
1.2	968	22.20	12.1	2.3
1.3	1102	22.80	12.9	2.2
1.4	1236	23.32	13.6	2.1
1.5	1370	23.76	14.3	2.0

9. The tabulation given above, and those which follow, are for illustrative purposes. The safety factor at the landside levee toe and the landside berm toe for design use are items that depend on judgement and will not be discussed in this report. The uplift safety factor at the landside levee toe should be such as to prevent slope failure under maximum riverside pool. Note that the safety factor is uniform throughout the width of the berm. However, variable safety factors may be used. For example, assume that $F = 1.5$ at the levee toe and 1.0 at the berm toe with $B = 700$ ft and $h_t = 20.65$ ft. Then by substituting in Equation 8

$$t = \frac{20.65 \times 1 \times 1.5}{1 + (1 \times 1.5)} = 12.4 \text{ ft}$$

Case II - Infinitely Pervious Berm

10. This case can be approached in theory but not realized in design; however, it is closely approached when the top blanket permeability is very much smaller than that of the berm (an order of 1 to 500). The berm is assumed to be made of such pervious soil that no head loss occurs in the vertical or horizontal seepage at the base of the

berm. The equations describing the flow conditions in the top blanket were presented by Bennett.* The origin of the x coordinate is at the landside levee toe and is positive landward. The assumption of infinite horizontal permeability for the berm negates assumption e (see paragraph 2) for this case.

11. The uplift head at the base of the top blanket under the landside levee toe, h_t , is expressed as

$$h_t = \frac{HL_{LS}}{\underline{\overline{X}} + L_2 + L_{LS}} \tag{9}$$

12. The uplift head under the semipervious top blanket at the berm landside toe, h_a , is controlled by the uplift safety factor. The uplift head at the base of the top blanket, h_x at point x, landward of the landside levee toe is written as

$$h_x = h_t e^{-cx} \tag{10}$$

At $x = B$ (landside berm toe), $h = h_a$; by using Equation 2

$$h_a = \frac{Z_b \gamma'_b}{\gamma_w F}$$

then

$$-cB = \ln\left(\frac{Z_b \gamma'_b}{\gamma_w F h_t}\right) \tag{11}$$

By rearranging Equation 11, the berm width, B , becomes

---

* Preston T. Bennett. 1946. "The Effect of Blankets on Seepage Through Pervious Foundations," Transactions of the American Society of Civil Engineers, Vol III, pp 215-252.

$$B = \frac{1}{c} \ln \left(\frac{\gamma_w F h_t}{Z_b \gamma_b'} \right) \tag{12}$$

Assuming that the landside tailwater is just at the top of the semi-pervious top blanket, the safety factor against uplift, F_x at point x, is expressed as

$$F_x = \frac{Z_b \gamma_b' + t_x \gamma_m'}{h_x \gamma_w} \tag{13}$$

Then, the berm thickness, t_x at point x, can be written as

$$t_x = \frac{h_x \gamma_w F - Z_b \gamma_b}{\gamma_m'} \tag{14}$$

where γ_m' is the moist unit weight of the berm and h_x is obtained from Equation 10. If part of the berm is submerged, then that part requires the use of a buoyant weight in computing the berm thickness.

13. Using the data for the example given in Case I (see paragraph 5), a uniform safety factor and a moist unit weight, γ_m', that is twice as large as the buoyant unit weight of the top blanket, γ_b', and the unit weight of water, γ_w, the uplift under the landside levee toe, h_t, is determined by substituting in Equation 9.

$$h_t = \frac{30 \times 223.6}{218.5 + 200 + 223.6} = 10.45 \text{ ft}$$

Also,

$$h_a = \frac{5 \times 1}{1 \times F}$$

$$B = 223.6 \ln \left(\frac{1 \times F \times 10.45}{5 \times 1} \right)$$

$$\gamma_m' = 2\gamma_b' = 2\gamma_w$$

$$t = \frac{10.45 \times 1 \times F - (5 \times 1)}{2}$$

The berm thickness, t at $x = 0$, is given in the following tabulation.

Safety Factor F	Berm Length B, ft	Uplift at $x = B$ h_a, ft	Berm Thickness t at $x = 0$ ft
1.0	165	5.0	2.7
1.1	186	4.5	3.2
1.2	206	4.2	3.8
1.3	223	3.8	4.3
1.4	240	3.6	4.8
1.5	255	3.3	5.3

14. The berm thickness for the case where the uplift safety factor is 1.5 is determined by substituting in Equations 10 and 14.

$$h_x = 10.45e^{-x/223.6}$$

$$t_x = \frac{(h_x \times 1 \times 1.5) - (5 \times 1)}{2}$$

The following tabulation is obtained for h_x and t_x.

x, ft	h_x, ft	t_x, ft
0	10.45	5.3
50	8.36	3.8
100	6.68	2.5
150	5.34	1.5
200	4.27	0.7
255	3.34	0

Case III - Infinitely Pervious Berm in Vertical Direction

15. In this case, the horizontal permeability equals zero, and the safety factor is constant. The differential equation for seepage

upward through the top blanket and berm is written as

$$\frac{k_f D d^2 h}{dx^2} = \frac{k_b \left(h_x - t_x\right)}{Z_b} \tag{15}$$

It should be noted that only the semipervious top blanket has seepage resistance since the vertical permeability of the berm is infinite.

16. Equation 15 can be expressed as

$$\frac{d^2 h}{dx^2} = \frac{k_b}{k_f D Z_b} \left(h_x - t_x\right) = c^2 \left(h_x - t_x\right) \tag{16}$$

where t_x is the variable thickness of the berm. The seepage flows vertically in the berm (the horizontal permeability assumed to be zero). The berm has a buoyant unit weight, γ_t'.

17. The uplift safety factor of the combined seepage berm and top blanket is then

$$F = \frac{Z_b \gamma_b' + t_x \gamma_t'}{\left(h_x - t_x\right) \gamma_w} \tag{17}$$

and

$$h_x - t_x = \frac{Z_b \gamma_b' + t_x \gamma_t'}{\gamma_w F}$$

$$= \frac{Z_b \gamma_b'}{\gamma_w F} + \frac{t_x \gamma_t'}{\gamma_w F}$$

so that

$$h_x - \frac{Z_b \gamma_b'}{\gamma_w F} = t_x \left(1 + \frac{\gamma_t'}{\gamma_w F}\right) = t_x \left(\frac{\gamma_w F + \gamma_t'}{\gamma_w F}\right)$$

and

$$t_x = \left(\frac{h_x \gamma_w F}{\gamma_w F + \gamma_t'}\right) - \left(\frac{Z_b \gamma_b'}{\gamma_w F + \gamma_t'}\right) \tag{18}$$

where the berm thickness, t_x at point x, is expressed in terms of the uplift, h_x, at the base of the semipervious top blanket, the safety factor, unit weights, and top blanket thickness.

18. Set

$$\frac{\gamma_w F}{\gamma_w F + \gamma_t'} = \overline{A}$$

$$\frac{Z_b \gamma_b'}{\gamma_w F + \gamma_t'} = \overline{B}$$

$$\frac{k_b}{k_f D Z_b} = c^2$$

Then Equation 16 can be expressed as

$$\frac{d^2h}{dx^2} = c^2 (h_x - t_x) = c^2 (h_x - h_x\overline{A} + \overline{B})$$

$$= h_x c^2 (1 - \overline{A}) + c^2\overline{B} \tag{19}$$

$$= h_x \theta + \xi$$

where

$$c^2 (1 - \overline{A}) = \theta$$

and

$$c^2\overline{B} = \xi$$

Now set

$$h_x\theta + \xi = y$$

$$\theta \frac{dh}{dx} = \frac{dy}{dx}$$

$$\frac{d^2h}{dx^2} = \frac{1}{\theta}\left(\frac{dy^2}{dx^2}\right) = y \tag{20}$$

so that

$$\frac{d^2y}{dx^2} = \theta y \tag{21}$$

A solution to Equation 21 is

$$y = C_1 e^{x\sqrt{\theta}} + C_2 e^{-x\sqrt{\theta}} = \theta h_x + \xi \tag{22}$$

or

$$h_x = \frac{C_1}{\theta} e^{x\sqrt{\theta}} + \frac{C_2}{\theta} e^{-x\sqrt{\theta}} - \frac{\xi}{\theta} \tag{23}$$

The coordinate system is zero at the landward berm toe, and x is positive in the landward direction. At $x = 0$ (berm landward toe), $h = h_a$; thus

$$h_a = \frac{C_1}{\theta} + \frac{C_2}{\theta} - \frac{\xi}{\theta} \tag{24}$$

Now

$$\frac{\xi}{\theta} = \frac{c^2\overline{B}}{c^2\ (1 - \overline{A})} = \left(\frac{Z_b\gamma_b'}{\gamma_w F + \gamma_t'}\right)\left(\frac{\gamma_w F + \gamma_t'}{\gamma_t'}\right) \tag{25}$$

and so Equation 24 may be expressed as

$$\left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right) \theta = C_1 + C_2 \tag{26}$$

but

$$\theta = c^2 (1 - \overline{A}) = c^2 \left(\frac{\gamma_t'}{\gamma_w F + \gamma_t'}\right)$$

and Equation 26 becomes

$$\left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right)\left(\frac{c^2 \gamma_t'}{\gamma_w F + \gamma_t'}\right) \tag{27}$$

At $x = 0$

$$\frac{dh}{dx} = - \frac{h_a}{L_{LS}} = \frac{C_1 \sqrt{\theta}}{\theta} - \frac{C_2 \sqrt{\theta}}{\theta} \tag{28}$$

Thus, the two simultaneous equations that result are

$$C_1 - C_2 = - \frac{h_a \sqrt{\theta}}{L_{LS}} \tag{29}$$

$$C_1 + C_2 = \left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right) \theta \tag{30}$$

Then

$$C_1 = \left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right) \frac{\theta}{2} - \frac{h_a \sqrt{\theta}}{2L_{LS}} \tag{31}$$

and

$$C_2 = \left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right) \frac{\theta}{2} + \frac{h_a \sqrt{\theta}}{2L_{LS}} \tag{32}$$

so that Equation 23 becomes

$$h_x + \frac{Z_b \gamma_b'}{\gamma_t'} = \left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right) \frac{e^{x\sqrt{\theta}}}{2} - \frac{h_a \sqrt{\theta}}{2L_{LS}\theta} e^{x\sqrt{\theta}}$$

$$+ \left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right) \frac{e^{-x\sqrt{\theta}}}{2} + \frac{h_a \sqrt{\theta}}{2L_{LS}\theta} e^{-x\sqrt{\theta}}$$

$$= \left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right)\left(\frac{e^{x\sqrt{\theta}} + e^{-x\sqrt{\theta}}}{2}\right) - \frac{h_a}{L_{LS}\sqrt{\theta}} \left(\frac{e^{x\sqrt{\theta}} - e^{-x\sqrt{\theta}}}{2}\right)$$

$$= \left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right) \cosh\,(x\sqrt{\theta}) - \frac{h_a}{L_{LS}\sqrt{\theta}} \sinh\,(x\sqrt{\theta}) \tag{33}$$

At $x = -B$ (landside levee toe), $h = h_t$; thus

$$h_t + \frac{Z_b \gamma_b'}{\gamma_t'} = \left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right) \cosh\,(B\sqrt{\theta}) + \frac{h_a}{L_{LS}\sqrt{\theta}} \sinh\,(B\sqrt{\theta}) \tag{34}$$

Also, at $x = -B$

$$\frac{dh}{dx} = -\left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right) \sqrt{\theta} \sinh\,(B\sqrt{\theta}) - \frac{h_a}{L_{LS}} \cosh\,(B\sqrt{\theta})$$

$$= -\frac{H - h_t}{\underline{X} + L_2} \tag{35}$$

$$H - h_t = (\underline{X} + L_2)\ \sqrt{\theta} \left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right) \sinh\,(B\sqrt{\theta})$$

$$+ (\underline{X} + L_2)\ \frac{h_a}{L_{LS}} \cosh\,(B\sqrt{\theta}) \tag{36}$$

Now, substitute in Equation 34 to obtain

$$H - h_t = \left(H + \frac{Z_b\gamma_b'}{\gamma_t'}\right) - \left(h_a + \frac{Z_b\gamma_b'}{\gamma_t'}\right)\cosh\ (B\sqrt{\theta})$$

$$- \frac{h_a}{L_{LS}\sqrt{\theta}}\sinh\ (B\sqrt{\theta}) \tag{37}$$

Then equate Equations 36 and 37 to obtain

$$H + \frac{Z_b\gamma_b'}{\gamma_t'} = \cosh\ (B\sqrt{\theta})\left[(\overline{X} + L_2)\frac{h_a}{L_{LS}} + \left(h_a + \frac{Z_b\gamma_b'}{\gamma_t'}\right)\right]$$

$$+ \sinh\ (B\sqrt{\theta})\left[\left(h_a + \frac{Z_b\gamma_b'}{\gamma_t'}\right)(\overline{X} + L_2)\sqrt{\theta} \right. \tag{38}$$

$$\left. + \frac{h_a}{L_{LS}\sqrt{\theta}}\right]$$

Now

$$h_a = \frac{Z_b\gamma_b'}{\gamma_w F}$$

and

$$\sqrt{\theta} = c\sqrt{\frac{\gamma_t'}{\gamma_t' + \gamma_w F}}$$

so that

$$H + \frac{Z_b\gamma_b'}{\gamma_t'} = \bar{\bar{A}}\cosh\ (B\sqrt{\theta}) + \bar{\bar{B}}\sinh\ (B\sqrt{\theta}) \tag{39}$$

where

$$\bar{\bar{A}} = \frac{Z_b\gamma_b'}{\gamma_t'} + h_a\left(\frac{\overline{X} + L_2 + L_{LS}}{L_{LS}}\right)$$

$$\bar{\bar{B}} = \left(h_a + \frac{Z_b \gamma'_b}{\gamma'_t}\right) (\underline{X} + L_2) \sqrt{\theta} + \frac{h_a}{L_{LS}\sqrt{\theta}}$$

and so

$$\left(H + \frac{Z_b \gamma'_b}{\gamma'_t}\right) - \bar{\bar{A}} \cosh (B\sqrt{\theta}) - \bar{\bar{B}} \sinh (B\sqrt{\theta})$$

$$= \left(H + \frac{Z_b \gamma'_b}{\gamma'_t}\right) - \frac{\bar{\bar{A}}}{2} \left(e^{B\sqrt{\theta}} + e^{-B\sqrt{\theta}}\right) - \frac{\bar{\bar{B}}}{2} \left(e^{B\sqrt{\theta}} - e^{-B\sqrt{\theta}}\right)$$

$$= \left(H + \frac{Z_b \gamma'_b}{\gamma'_t}\right) - \left(\frac{\bar{\bar{A}} + \bar{\bar{B}}}{2}\right) e^{B\sqrt{\theta}} - \left(\frac{\bar{\bar{A}} - \bar{\bar{B}}}{2}\right) e^{-B\sqrt{\theta}} = \delta \qquad (40)$$

For any given set of conditions various values of berm length, B , are tried until δ is equal to zero.

Example: (same as for Case I)

$$\sqrt{\theta} = \frac{1}{\sqrt{200 \times 50 \times 5}} \times \frac{1}{\sqrt{1 + F}}$$

$$= \frac{1}{\sqrt{50,000}} \times \frac{1}{\sqrt{1 + F}}$$

$$= \frac{1}{223.6} \times \frac{1}{\sqrt{1 + F}}$$

The following tabulations illustrate Case III, for the infinitely pervious berm in the vertical direction.

Safety Factor F	h_a, ft	10^3 $(\sqrt{\theta})$	$\frac{h_a}{L_{LS}\sqrt{\theta}}$	$h_a + Z_b$	$(h_a + Z_b) \times (\underline{X} + L_2) \sqrt{\theta}$
1.0	5.0000	3.162	7.0711	10.000	13.2330
1.1	4.5455	3.086	6.5870	9.545	12.3273
1.2	4.1667	3.015	6.1802	9.167	11.5667
1.3	3.8462	2.949	5.8330	8.846	10.9173
1.4	3.5714	2.887	5.5328	8.571	10.3556
1.5	3.3333	2.828	5.2705	8.333	9.8623

Safety Factor F	$\bar{\bar{B}}$	$\bar{\bar{A}}$	$\bar{\bar{A}} + \bar{\bar{B}}$	$-\bar{\bar{A}} + \bar{\bar{B}}$
1.0	20.3041	19.3582	39.6623	0.9459
1.1	18.9143	18.0529	36.9672	0.8614
1.2	17.7469	16.9652	34.7121	0.7817
1.3	16.7503	16.0448	32.7951	0.7055
1.4	15.8884	15.2559	31.1443	0.6325
1.5	15.1328	14.5722	29.7050	0.5606

$\dfrac{\bar{\bar{A}} - \bar{\bar{B}}}{2} e^{-B\sqrt{\theta}}$ is rather small. If it is neglected and $\delta = 0$, then

$$\left(H + \frac{Z_b \gamma_b'}{\gamma_t'}\right) \frac{2}{\bar{\bar{A}} + \bar{\bar{B}}} = e^{B\sqrt{\theta}} \tag{41}$$

and so an approximation of the berm length is

$$B = \frac{1}{\sqrt{\theta}} \ln \left[\left(H + \frac{Z_b \gamma_b'}{\gamma_t'}\right) \frac{2}{\bar{\bar{A}} + \bar{\bar{B}}}\right] \tag{42}$$

as shown in the following tabulation.

F	B , ft Eg (40)	B_{APPROX} , ft Eg (42)
1.0	182	180
1.1	209	207
1.2	234	233
1.3	259	257
1.4	282	281
1.5	304	303

19. The uplift, h_x , at the base of the top, semipervious blanket can be obtained from Equation 33. (<u>Note that x is negative and that $x = 0$ is at the landside berm toe.</u>)

$$h_x = \left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right) \cosh\,(x\sqrt{\theta}) - \frac{h_a}{L_{LS}\sqrt{\theta}} \sinh\,(x\sqrt{\theta}) - \frac{Z_b \gamma_b'}{\gamma_t'} \tag{43}$$

The berm thickness at point $-x$ can be obtained from Equation 17 so that

$$t_x = \frac{h\gamma_w F - Z_b \gamma_b'}{\gamma_w F + \gamma_t'} \tag{44}$$

For a safety factor equal to 1.5 and noting that x is negative, the following tabulation for t_x represents the data from the example above.

x , ft	h_x , ft	t_x , ft
0	$3.33 = h_x$	0.0
-50	4.16	0.5
-100	5.18	1.1
-150	6.40	1.8
-200	7.84	2.7
-250	9.55	3.7
-304	$11.72 = h_t$	5.0

Case IV - Permeability of Seepage Berm Equal to That of Top Blanket

20. The origin of the x coordinate is taken at the landside seepage berm toe with a positive direction towards the river. The factor of safety against seepage uplift is assumed to be a constant F (Equation 17). The seepage gradient, i , is expressed as

$$i = \frac{h_x - t_x}{t_x + Z_b} \tag{45}$$

The differential equation relating the flow up through the combined top blanket and seepage berm to that flowing horizontally through the pervious foundation (note that $k_b = k_t$) is

$$k_f D \frac{d^2h}{dx^2} = k_b \left(\frac{h_x - t_x}{t_x + Z_b} \right) = k_b i \tag{46}$$

If the buoyant unit weight of the top blanket, γ_b' , is equal to that of the seepage berm, γ_t' , then the safety factor of the combined berm and top blanket is written as

$$F = \frac{\gamma_b' \ (t_x + Z_b)}{\gamma_w \ (h_x - t_x)} \tag{47}$$

Thus, Equation 46 can be rewritten as

$$\frac{d^2h}{dx^2} = \frac{k_b\gamma_b'}{D\gamma_w F} = \theta_1 \tag{48}$$

If the buoyant weight of the berm is less than that of the top blanket ($\gamma_t' < \gamma_b'$), then with a constant seepage gradient upward through the top blanket and berm ($k_b = k_t$), the uplift safety factor for the berm only is

$$F = \left(\frac{Z_b + t_x}{h_x - t_x}\right)\frac{\gamma_t'}{\gamma_w} \tag{49}$$

and

$$\frac{h_x - t_x}{Z_b + t_x} = \frac{\gamma_t'}{\gamma_w F} \tag{50}$$

so that

$$\frac{d^2h}{dx^2} = \frac{k_b\gamma_t'}{k_f D\gamma_w F} = \theta_2 \tag{51}$$

where θ_1 and θ_2 are constants such that

$$\theta_2 = \theta_1 \frac{\gamma_t'}{\gamma_b'}$$

The solution of Equation 48 is

$$\frac{dh}{dx} = \theta_1 x + C_1 \tag{52}$$

At $x = 0$

$$\frac{dh}{dx} = C_1 = \frac{h_a}{L_{LS}}$$

where L_{LS} is the effective length of the top blanket landward of the seepage berm landside toe. Thus

$$\frac{dh}{dx} = \theta_1 x + \frac{h_a}{L_{LS}} \tag{53}$$

Integrating again

$$h_x = \theta_1 \frac{x^2}{2} + \frac{h_a x}{L_{LS}} + C_2 \tag{54}$$

At $x = 0$, $h = h_a = C_2$ so that

$$h_x = \theta_1 \frac{x^2}{2} + \frac{h_a x}{L_{LS}} + h_a \tag{55}$$

and by inserting the values of θ_1 and h_a in Equation 55

$$h_x = \frac{k_b \gamma_b' x^2}{2k_f D \gamma_w F} + \frac{Z_b \gamma_b' x}{\gamma_w F L_{LS}} + \frac{Z_b \gamma_b'}{\gamma_w F} \tag{56}$$

At $x = B$, $h = h_t$ so that

$$h_t = \frac{k_b \gamma_b' B^2}{2k_f D \gamma_w F} + \frac{Z_b \gamma_b' B}{\gamma_w F L_{LS}} + \frac{Z_b \gamma_b'}{\gamma_w F} \tag{57}$$

Now at $x = B$, the slope of the piezometric profile is

$$\frac{dh_x}{dx} = \frac{k_b \gamma_b' B}{k_f D \gamma_w F} + \frac{Z_b \gamma_b'}{\gamma_w F L_{LS}} = \frac{H - h_t}{\overline{X} + L_2} \tag{58}$$

so that the uplift head, h_t , is

$$h_t = H - \frac{k_b \gamma_b' B\ (\overline{X} + L_2)}{k_f D \gamma_w F} - \frac{Z_b \gamma_b'\ (\overline{X} + L_2)}{\gamma_w F L_{LS}} \tag{59}$$

By equating the expression above and the Equation 57 the value of B may be written as

$$B = (\overline{X} + L_2 + L_{LS}) \left[-1 + \sqrt{1 + \frac{2H\gamma_w F L_{LS}^2}{\gamma_b' Z_b \, (\overline{X} + L_2 + L_{LS})^2} - \frac{2L_{LS}}{\overline{X} + L_2 + L_{LS}}} \right] \tag{60}$$

If the buoyant weight of the seepage berm is less than that of the top blanket, the expression θ_2 should be used rather than θ_1. This requires the use of γ_t' in Equation 60 rather than γ_b'. Use the data from the Case I example (see paragraph 5) to obtain

$$B = 642.1 \left(-1 + \sqrt{1.45518934F + 0.3035352749} \right)$$

as shown in the following tabulation.

Safety Factor F	Berm Length B , ft
1.0	209
1.1	244
1.2	277
1.3	309
1.4	340
1.5	370

21. The uplift at the base of the top semipervious blanket, h_x, referred to the upper surface of the blanket may be obtained using Equation 56. The thickness of the seepage berm, t_x at point x, may be computed as indicated below. The uplift safety factor, F , is given by Equation 17 so that

$$t_x = \frac{h_x - \dfrac{Z_b \gamma_b'}{\gamma_w F}}{1 + \dfrac{\gamma_b'}{\gamma_w F}} \tag{61}$$

22. Use the data from the Case I example with the safety $F = 1.2$ and $B = 277$ ft. Then Equation 56 becomes

$$h_x = \frac{1 \times x^2}{2 \times 1 \times 1.2 \times 200 \times 50} + \frac{5 \times 1 \times x}{1 \times 1.2 \times 223.6} + \frac{5 \times 1}{1 \times 1.2}$$

and Equation 61 is

$$t_x = \frac{h_x - \frac{5 \times 1}{1 \times 1.2}}{\frac{1}{1 \times 1.2} + 1} = \frac{1.2h_x - 5}{1.2 + 1} = \frac{6h_x - 25}{11}$$

as shown in the following tabulation.

x, ft	h_x, ft	t_x, ft
0	4.167	0.0
50	5.203	0.6
100	6.447	1.2
150	7.899	2.0
200	9.560	2.9
250	11.429	4.0
277	12.525	4.6

Case V - Semipervious Seepage Berm

23. It is assumed for this case that the vertical permeability of the seepage berm, k_t, is equal to or less than that of the semipervious top blanket and that the horizontal permeabilities of both the seepage berm and semipervious top blanket are zero. The x coordinate system is zero at the landside toe of the seepage berm, and the positive direction is riverward.

24. When seepage flows perpendicular to soil layers 1 and 2, it is helpful to express the thickness of one layer so that it has a thickness-permeability relationship such that a common permeability coefficient may be used. Thus, if layer 1 has k_1 and Z_1 and layer 2

has k_2 and Z_2 , then the equivalent thickness of Z_2 , using k_1 in lieu of k_2 , is

$$Z_2' = \frac{Z_2 k_1}{k_2}$$

or the equivalent thickness of layer 1, using k_2 for both layers, is

$$Z_1' = \frac{Z_1 k_2}{k_1}$$

The thickness of the top blanket, Z_b , can be expressed as Z_b' , to permit the use of the seepage berm permeability.

$$Z_b' = \frac{Z_b k_t}{k_b} \tag{62}$$

The differential equation for this case is

$$k_f D \frac{d^2 h}{dx^2} = \frac{k_t \ (h_x - t_x)}{Z_b' + t_x} \tag{63}$$

Rewrite Equation 63 so that

$$\frac{d^2 h}{dx^2} = \frac{k_t}{k_f D} \left(\frac{h_x - t_x}{\dfrac{Z_b k_t}{k_b} + t_x} \right) \tag{64}$$

The seepage gradient up through the berm and the equivalent top blanket is

$$i = \frac{h_x - t_x}{t_x + \dfrac{k_t Z_b}{k_b}} \tag{65}$$

The uplift safety factor in the <u>seepage berm only</u> is a constant F . Then

$$F = \frac{\gamma_t'}{\gamma_w i} \tag{66}$$

where

$$i = \frac{\gamma_t'}{\gamma_w F}$$

so that

$$\frac{d^2h}{dx^2} = \frac{k_t i}{k_f D} = \frac{k_t \gamma_t'}{k_f D \gamma_w F} = \theta \tag{67}$$

Integrating

$$\frac{dh}{dx} = \theta x + C_1 \tag{68}$$

$$h_x = \frac{\theta x^2}{2} + C_1 x + C_2 \tag{69}$$

At $x = 0$, $h = h_a = C_2$ where h_a is the uplift head just under the semipervious top blanket. At $x = 0$ (using Equations 1 and 2)

$$F = \frac{\gamma_b' Z_b}{\gamma_w h_a}$$

$$h_a = C_2 = \frac{\gamma_b' Z_b}{\gamma_w F}$$

Also at $x = 0$, the landside seepage gradient is

$$\frac{dh}{dx} = \frac{h_a}{L_{LS}} \tag{70}$$

where L_{LS} is the effective landside length of the top blanket measured landward from $x = 0$. Then at $x = 0$

$$\frac{dh}{dx} = \theta \times 0 + C_1 = \frac{h_a}{L_{LS}} = \frac{Z_b \gamma_b'}{\gamma_w F L_{LS}} \tag{71}$$

so that the uplift head at point x at the base of the top blanket is

$$h_x = \frac{k_t \gamma_t' x^2}{2k_f D \gamma_w F} + \frac{Z_b \gamma_b' x}{\gamma_w F L_{LS}} + \frac{Z_b \gamma_b'}{\gamma_w F} \tag{72}$$

At $x = B$ (landside levee toe), the seepage uplift at the base of the top blanket is

$$h_t = \frac{k_t \gamma_t' B^2}{2k_f D \gamma_w F} + \frac{Z_b \gamma_b' B}{\gamma_w F L_{LS}} + \frac{Z_b \gamma_b'}{\gamma_w F} \tag{73}$$

Also at $x = B$, the seepage gradient in the pervious foundation is

$$\frac{H - h_t}{\overline{X} + L_2} = \frac{k_t \gamma_t' B}{k_f D \gamma_w F} + \frac{Z_b \gamma_b'}{\gamma_w F L_{LS}} \tag{74}$$

and so

$$h_t = H - \frac{Z_b \gamma_b' (\overline{X} + L_2)}{\gamma_w F L_{LS}} - \frac{k_t \gamma_t' B (\overline{X} + L_2)}{\gamma_w D F k_f} \tag{75}$$

Equate Equations 73 and 75 to obtain

$$B = \left(\frac{k_b \gamma_b' L_{LS}}{k_t \gamma_t'} + \overline{X} + L_2 \right) \left[-1 + \sqrt{1 + (\ )} \right] \tag{76}$$

where

$$(\ ) = \frac{\left[2 \quad \frac{H \gamma_w F k_b}{\gamma_t' z_b k_t} - \frac{\gamma_b' k_b}{\gamma_t' k_t} \left(\frac{\overline{X} + L_2 + L_{LS}}{L_{LS}} \right) \right]}{\left(\frac{\gamma_b' k_b}{\gamma_t' k_t} + \frac{\overline{X} + L_2}{L_{LS}} \right)} \tag{77}$$

If $\gamma_b' = \gamma_t'$ and $k_b = k_t$, Equation 76 becomes Equation 60.

25. Use data from the Case I example (see paragraph 5) to obtain the following tabulation with

$$\overline{K} = \frac{k_t}{k_b}$$

where $\overline{K}$ equals the ratio of the seepage berm permeability to that of the top blanket.

$\overline{K}$	Seepage Berm Width, B , ft F = 1.0	F = 1.1	F = 1.2	F = 1.3	F = 1.4	F = 1.5
1	209	244	277	309	340	370
0.2	445	519	591	600	727	793
0.1	535	628	718	806	892	975
0.05	603	711	817	921	1023	1124
0.02	656	778	898	1018	1136	1253
0.01	677	804	931	1057	1183	1308
0.001	697	831	964	1097	1231	1364
0.0001	699	833	967	1102	1236	1370
0.00001	700	834	968	1102	1236	1370

26. For the small values of $\overline{K}$, the values of B from the tabulation above are equal to those in the tabulation on page 11; when $\overline{K}$ is unity, the values of B equals those in the tabulation on page 26. Using a concept introduced by Polubarinova-Kochina, write

$$\tan(\pi\varepsilon) = \sqrt{\frac{k_b}{k_t}} = \frac{1}{\sqrt{\overline{K}}} \tag{78}$$

so that

$$\varepsilon = \frac{1}{\pi} \text{ arc tan } \frac{1}{\sqrt{\overline{K}}} \tag{79}$$

as shown in the following tabulation.

$\overline{K}$	ε
Infinity	0
10	0.097
5	0.134
1	0.250
0.2	0.366
0.1	0.403
0.05	0.430
0.01	0.468
0	0.500

27. Having selected an uplift safety factor, F, for the seepage berm and having determined the berm length, B, the uplift head, h_x, at the base of the semipervious top blanket, (referenced to the upper surface of the blanket) can be determined using Equation 72. The seepage head at the base of the seepage berm, h'_x, (or the top of the semipervious top blanket) is found as follows:

$$\frac{h_x - t_x}{t_x + Z_b \overline{K}} = \frac{h'_x - t_x}{t_x} \tag{80}$$

so that

$$h'_x - t_x = \frac{h_x - t_x}{1 + \frac{Z_b \overline{K}}{t_x}} \tag{81}$$

The uplift safety factor for the seepage berm is a constant F. Then

$$F = \frac{t_x \gamma'_t}{(h'_x - t_x)\,\gamma_w} \tag{82}$$

so that

$$h'_x - t_x = \frac{t_x \gamma'_t}{\gamma_w F} \tag{83}$$

28. Equate Equations 81 and 83 to obtain the following expression for the berm thickness at point x

$$t_x = \frac{h_x - \frac{Z_b \gamma'_t \overline{K}}{\gamma_w F}}{1 + \frac{\gamma'_t}{\gamma_w F}} \tag{84}$$

Using the data from the example presented in Case I with $F = 1.2$, $\overline{K} = 1.0$, and $B = 277$ ft, Equation 72 is

$$h_x = \left(\frac{1}{200 \times 50 \times 1 \times 1.2 \times 2}\right) x^2 + \left(\frac{1 \times 5}{1 \times 1.2 \times 223.6}\right) x + \frac{1 \times 5}{1 \times 1.2}$$

and

$$t_x = \frac{h_x - \frac{5 \times 1 \times 1}{1 \times 1.2}}{1 + \frac{1}{1 \times 1.2}} = \frac{1.2h_x - 5}{2.2}$$

$$= \frac{6h_x}{11} - \frac{25}{11}$$

as given in the following tabulation, which agrees with values in the tabulation on page 27.

x, ft	h_x, ft	t_x, ft
0	4.167	0.0
50	5.203	0.6
100	6.447	1.2
150	7.899	2.0
200	9.560	2.9
250	11.429	4.0
277	12.525	4.6

29. As another example, assume that $F = 1.2$, $\overline{K} = 0.00001$, and $B = 968$ ft. Then

$$h_x = \left(\frac{1}{100{,}000 \times 200 \times 50 \times 1 \times 1.2 \times 2}\right) x^2$$

$$+ \left(\frac{1 \times 5}{1 \times 1.2 \times 223.6}\right) x + \frac{1 \times 5}{1 \times 1.2}$$

$$t_x = \frac{h_x - \dfrac{5 \times 1 \times 1}{1 \times 100{,}000 \times 1.2}}{1 + \dfrac{1}{1 \times 1.2}}$$

$$= \frac{1.2h_x}{2.2} - \frac{5}{2.2 \times 100{,}000}$$

$$= \frac{6h_x}{11} - \frac{25}{1{,}100{,}000} \approx \frac{6h_x}{11}$$

The following tabulation is obtained for h_x and t_x (see tabulation on page 11).

x , ft	h_x , ft	t_x , ft
0	4.167	2.3
100	6.030	3.3
200	7.894	4.3
300	9.757	5.3
400	11.621	6.3
500	13.484	7.4
600	15.348	8.4
700	17.211	9.4
800	19.075	10.4
900	20.938	11.4
968	22.206	12.1

30. The results of the examples given in the two previous tabulations (see page 31 (bottom) and the one above) represent the extremes for $\overline{K} = 1$ and $\overline{K}$ equal to a very small value. A third case is given where $F = 1.2$, $\overline{K} = 0.1$, and $B = 718$ ft . Then

$$h_x = \left(\frac{1}{10 \times 200 \times 50 \times 1 \times 1.2 \times 2}\right) x^2 + \left(\frac{1 \times 5}{1 \times 1.2 \times 223.6}\right) x + \frac{1 \times 5}{1 \times 1.2}$$

$$t_x \doteq \frac{h_x - \dfrac{5 \times 1 \times 1}{1 \times 10 \times 1.2}}{1 + \dfrac{1}{1 \times 1.2}} = \frac{1.2h_x}{2.2} - \frac{5}{2.2 \times 10} = \frac{6h_x}{11} - \frac{2.5}{11}$$

with h_x and t_x given in the following tabulation.

x , ft	h_x , ft	t_x , ft
0	4.167	2.046
100	6.072	3.084
200	8.060	4.169
300	10.132	5.299
400	12.287	6.475
500	14.526	7.696
600	16.847	8.962
718	19.694	10.515

31. The development above is for the case where the uplift safety factor of the seepage berm only is considered. If the uplift safety factor of the combined seepage berm and semipervious top blanket is developed, then the basic differential equation is that given by Equation 119 which can be solved using Equation 121, the finite difference method. Setting $\overline{K} = k_t/k_b$, the uplift safety factor of the seepage berm, F_t , and that for the combined seepage berm and the semipervious top blanket, F_{t+b} , then becomes

$$\frac{F_t}{F_{t+b}} = \frac{t_x + Z_b\overline{K}}{t_x + \dfrac{Z_b\gamma_b'}{\gamma_t'}}$$

If

$$\overline{K} \gtrless \frac{\gamma_b'}{\gamma_t'}$$

then

$$F_t \gtrless F_{t+b}$$

Case VI - Variable Uplift Safety Factor

32. For this case, it is assumed that the uplift factor is F_o at the landside levee toe and varies in a linear manner to a lower value F_B at the landside berm toe. The x coordinate system has its origin at the levee toe and is positive landward. The berm width is B. The uplift safety factor at point x is as follows: If $k_t \leq k_b$, then the basic differential equation (Equation 64) is

$$\frac{d^2h}{dx^2} = \frac{k_t \left(h_x - t_x\right)}{k_f D \left(\frac{Z_b k_t}{k_b} + t_x\right)}$$

The upward seepage through the top semipervious blanket and seepage berm has a gradient (Equation 65) in the berm of

$$i = \frac{h_x - t_x}{t_x + \frac{k_t Z_b}{k_b}}$$

The seepage uplift safety factor for the berm only is

$$F = \frac{\gamma_t'}{\gamma_w i} = F_o - \left(F_o - F_B\right) \frac{x}{B} \tag{85}$$

Combining Equations 65 and 85

$$\frac{h_x - t_x}{t_x + \frac{k_t Z_b}{k_b}} = \frac{\gamma_t'}{\gamma_w} \left[\frac{1}{F_o - \left(F_o - F_B\right) \frac{x}{B}}\right] \tag{86}$$

and Equation 64 becomes

$$\frac{d^2h}{dx^2} = \frac{k_t \gamma_t'}{k_f D \gamma_w \left[F_o - \left(F_o - F_B\right) \frac{x}{B}\right]} \tag{87}$$

Set

$$\phi = F_o - \left(F_o - F_B\right) \frac{x}{B}$$

then

$$d\phi = -(F_o - F_B)\frac{dx}{B}$$

and

$$dx = \frac{-Bd\phi}{F_o - F_B}$$

so that

$$(dx)^2 = \frac{B^2(d\phi)^2}{(F_o - F_B)^2}$$

Equation 87 rewritten is

$$\frac{d^2h}{dx^2} = \frac{k_t\gamma_t'}{k_fD_f\gamma_w\phi} = \frac{d^2h\ (F_o - F_B)^2}{B^2d\phi^2} \tag{88}$$

and so

$$\frac{d^2h}{\partial\phi^2} = \left[\frac{k_t\gamma_t'B^2}{k_fD\gamma_w\ (F_o - F_B)^2}\right]\frac{1}{\phi} = \frac{\theta}{\phi} \tag{89}$$

where

$$\theta = \left[\frac{k_t\gamma_b'B^2}{k_fD\gamma_w\ (F_o - F_B)^2}\right] \tag{90}$$

Equation 89 can be integrated

$$\frac{dh}{d\phi} = \theta\ \ln\ \phi + C_1 \tag{91}$$

so that

$$\frac{dh}{d\phi} = -\frac{Bdh}{(F_o - F_B)\ dx} = \theta\ \ln\left[F_o - (F_o - F_B)\ \frac{x}{B}\right] + C_1 \tag{92}$$

and

$$\frac{dh}{dx} = -(F_o - F_B)\frac{\theta}{B}\ln\left[F_o - (F_o - F_B)\frac{x}{B}\right] - \frac{C_1(F_o - F_B)}{B} \tag{93}$$

at $x = B$, $\phi = F_B$ and $\frac{dh}{dx} = -\frac{h_a}{L_{LS}}$, then

$$C_1 = \frac{h_a B}{L_{LS}(F_o - F_B)} - \theta \ln F_B \tag{94}$$

Equation 91 can be reexpressed as

$$\frac{dh}{d\phi} = \theta \ln \phi + \frac{h_a B}{L_{LS}(F_o - F_B)} - \theta \ln F_B \tag{95}$$

which, upon integration, is

$$h_x = \theta\left[\phi \ln(\phi) - \phi - \phi \ln(F_B)\right] + \frac{h_a B\phi}{L_{LS}(F_o - F_B)} + C_2 \tag{96}$$

At $x = B$, $\phi = F_B$ and $h = h_a$, then

$$h_a = \theta\left[F_B \ln(F_B) - F_B - F_B \ln(F_B)\right] + \frac{h_a BF_B}{L_{LS}(F_o - F_B)} + C_2 \tag{97}$$

so that

$$C_2 = h_a + F_B\theta - \frac{h_a BF_B}{L_{LS}(F_o - F_B)} \tag{98}$$

Thus, Equation 96 becomes

$$h_x = \theta\left[F_o \ln\left(\frac{F_o}{F_B}\right) - F_o + F_B\right] + \frac{h_a B(\phi - F_B)}{L_{LS}(F_o - F_B)} + h_a \tag{99}$$

At $x = 0$, $\phi = F_o$ and $h = h_t$, then

$$h_t = \theta\left[F_o \ln\left(\frac{F_o}{F_B}\right) - F_o + F_B\right] + h_a\left(\frac{B + L_{LS}}{L_{LS}}\right) \tag{100}$$

Also at $x = 0$

$$\frac{dh}{dx} = -\frac{H - h_t}{\underline{\bar{X}} + L_2}$$

Now, substitute Equation 94 into Equation 93 to obtain

$$\frac{dh}{dx} = -(F_o - F_B)\frac{\theta}{B}\ln\left\{\left[F_o - (F_o - F_B)\frac{x}{B}\right]\frac{1}{F_B}\right\} - \frac{h_a}{L_{LS}} \tag{101}$$

At $x = 0$, $F_o - (F_o - F_B)\frac{x}{B} = F_o$ and $h_x = h_t$, then

$$\frac{dh}{dx} = -\frac{H - h_t}{\underline{\bar{X}} + L_2} = -\left[(F_o - F_B)\frac{\theta}{B}\ln\left(\frac{F_o}{F_B}\right)\right] - \frac{h_a}{L_{LS}} \tag{102}$$

and h_t may be expressed as

$$h_t = H - (\underline{\bar{X}} + L_2)(F_o - F_B)\frac{\theta}{B}\ln\left(\frac{F_o}{F_B}\right) - (\underline{\bar{X}} + L_2)\frac{h_a}{L_{LS}} \tag{103}$$

Equating Equations 100 and 103 results in

$$\frac{k_t\gamma_t' B^2}{k_f D\gamma_w\,(F_o - F_B)^2}\left[F_o\ln\left(\frac{F_o}{F_B}\right) - F_o + F_B\right] + \frac{h_a\,(B + L_{LS})}{L_{LS}}$$
$$= H - (\underline{\bar{X}} + L_2)\left[\frac{k_t\gamma_t' B}{k_f D\gamma_w\,(F_o - F_B)}\right]\ln\left(\frac{F_o}{F_B}\right)$$
$$- (\underline{\bar{X}} + L_2)\frac{h_a}{L_{LS}} \tag{104}$$

At $x = B$

$$F_B = \frac{Z_b\gamma_b'}{h_a\gamma_w} \quad \text{and} \quad h_a = \frac{Z_b\gamma_b'}{\gamma_w F_B}$$

Equation 104 may be expressed as

$$B^2\left[\frac{F_o}{(F_o - F_B)^2}\ln\left(\frac{F_o}{F_B}\right) - \frac{1}{F_o - F_B}\right] + B\left[\frac{\overline{X} + L_2}{F_o - F_B}\ln\left(\frac{F_o}{F_B}\right)\right.$$

$$\left. + \frac{L_{LS}\gamma_b' k_b}{F_B\gamma_t' k_t}\right] + \left[\frac{\gamma_b' k_b L_{LS}}{\gamma_t' k_t F_B}(\overline{X} + L_2 + L_{LS}) - \frac{HL_{LS}^2\gamma_w k_b}{Z_b\gamma_t' k_t}\right] = 0 \qquad (105)$$

33. Set

$$\overline{A} = \left[\frac{F_o}{(F_o - F_B)^2}\ln\left(\frac{F_o}{F_B}\right) - \frac{1}{F_o - F_B}\right]$$

$$\overline{C} = \left[\frac{\overline{X} + L_2}{F_o - F_B}\ln\left(\frac{F_o}{F_B}\right) + \frac{\gamma_b L_{LS}}{F_B\gamma_t'\overline{K}}\right]$$

$$\overline{D} = \left[\frac{\gamma_b' L_{LS}}{\gamma_t' F_B\overline{K}}(\overline{X} + L_2 + L_{LS}) - \frac{HL_{LS}^2\gamma_w}{Z_b\gamma_t'\overline{K}}\right]$$

Then Equation 105 can be expressed as

$$B^2\overline{A} + B\overline{C} + \overline{D} = 0 \qquad (106)$$

The solution for the berm width, B, is

$$B = \frac{\overline{C}}{2\overline{A}}\left(-1 + \sqrt{1 - \frac{4\overline{A}\overline{D}}{\overline{C}^2}}\right) \qquad (107)$$

Using data previously presented in Case I example (see paragraph 5) with $F_B = 1$ and $\overline{K} = k_t/k_b$, the berm lengths for various $\overline{K}$ and the safety factors at the levee toe are as shown in the following tabulation.

Safety Factor F_o	Berm Length, B, ft					
	$\overline{K} = 1$	$\overline{K} = 0.2$	$\overline{K} = 0.1$	$\overline{K} = 0.01$	$\overline{K} = 0.001$	$\overline{K} = 0.0001$
1.000001	209.4	444.6	535.3	676.6	697.1	699.3
1.1	215.2					

(Continued)

Safety Factor F_o	Berm Length, B, ft					
	$\bar{K} = 1$	$\bar{K} = 0.2$	$\bar{K} = 0.1$	$\bar{K} = 0.01$	$\bar{K} = 0.001$	$\bar{K} = 0.0001$
1.2	220.7					
1.3	255.8					
1.4	230.3					
1.5	235.3	470.8	555.5	680.2	697.5	699.3
3	286.3					
5	329.4	548.7	611.2	688.9	698.4	699.4
10	392.4					
100	590.6					
1000	678.5					
10,000	696.5					
100,000	699.1					
1,000,000	699.5	699.5	699.5	699.5	699.5	699.5

From Equation 86 the expression for the variation in the thickness, t_x, of the seepage berm is written as

$$h_x - t_x = \frac{\gamma_t'}{\gamma_w}\left[\frac{t_x + Z_b\bar{K}}{F_o - (F_o - F_B)\,\frac{x}{B}}\right]$$

$$= \frac{\gamma_t' t_x}{\gamma_w\left[F_o - (F_o - F_B)\,\frac{x}{B}\right]} + \frac{\gamma_t' Z_b\bar{K}}{\gamma_w\left[F_o - (F_o - F_B)\,\frac{x}{B}\right]} \qquad (108)$$

$$h_x - \frac{\gamma_t' Z_b\bar{K}}{\gamma_w\left[F_o - (F_o - F_B)\right]\frac{x}{B}} = t_x\left\{1 - \frac{\gamma_t'}{\gamma_w\left[F_o - (F_o - F_B)\,\frac{x}{B}\right]}\right\} \qquad (109)$$

$$t_x = \frac{h_x - \dfrac{\gamma_t' Z_b\bar{K}}{\gamma_w\left[F_o - (F_o - F_B)\,\frac{x}{B}\right]}}{\left(1 + \dfrac{\gamma_t'}{\gamma_w}\right)\left[\dfrac{1}{F_o - (F_o - F_B)\,\frac{x}{B}}\right]} \qquad (110)$$

The value of h_x at point x is obtained from Equation 99. Then

$$h_x = \frac{\bar{K}\gamma_t' B^2 k_b}{k_f D\gamma_w \; (F_o - F_B)^2} \left\{ \left[F_o - (F_o - F_B)\ \frac{x}{B}\right] \frac{x}{B} \right.$$

$$\times \ln \frac{\left[F_o - (F_o - F_B)\ \frac{x}{B}\right]}{F_B} - \left[F_o - (F_o - F_B)\ \frac{x}{B}\right] + F_B \Bigg\}$$

$$+ \frac{Z_b \gamma_b'}{\gamma_w F_B} \left(1 + \frac{B\left\{\left[F_o - (F_o - F_B)\ \frac{x}{B}\right] - F_B\right\}}{L_{LS}\ (F_o - F_B)}\right) \qquad (111)$$

Using the same data as given in Case I example $F_o = 1.5$, $F_B = 1.0$, $B = 235.3$, and $\bar{K} = k_t/k_b = 1.0$, Equation 110 becomes

$$t_x = \frac{h_x - \dfrac{5}{1.5 - \dfrac{0.5x}{235.3}}}{1 + \dfrac{1}{1.5 - \dfrac{0.5x}{235.3}}} = \frac{h_x\left(1.5 - \dfrac{0.5x}{235.3}\right) - 5}{\left(1.5 - \dfrac{0.5x}{235.3}\right) + 1}$$

and Equation 111 is

$$h_x = 5 \left\{ \left(\frac{235.3}{223.6 \times 0.5}\right)^2 \left[\left(1.5 - \frac{0.5x}{235.3}\right) \ln \left(1.5 - \frac{0.5x}{235.3}\right)\right.\right.$$

$$\left.\left. - \left(1.5 - \frac{0.5x}{235.3}\right) + 1\right]\right\} + 5 \left[1 + \frac{235.3 \left(1.5 - \dfrac{0.5x}{235.3}\right) - 1}{223.6\ (1.5 - 1)}\right]$$

$$= 5 \left\{ 4.429556517 \left[\left(1.5 - \frac{x}{470.6}\right) \ln \left(1.5 - \frac{x}{470.6}\right)\right.\right.$$

$$\left.\left. - \left(1.5 - \frac{x}{470.6}\right) + 1\right]\right\} + 5 \left[1 + \frac{235.3 \left(1.5 - \dfrac{x}{470.6}\right) - 1}{111.8}\right]$$

With values of h_x and t_x given in the following tabulation.

x , ft	h_x , ft	t_x , ft
0	12.658	5.59
50	10.671	4.12
100	8.864	2.80
150	7.251	1.63
200	5.850	0.62
235.3	5.000	0.00

Case VII - Pervious Seepage Berm

34. For this case, the vertical permeability of the berm is equal to or larger than that of the top, semipervious blanket. As with most of the other cases, the horizontal permeability of the seepage blanket is assumed to be zero. The origin of the x coordinate system is at the landside toe of the seepage berm and is positive towards the river. The basic differential equation is

$$k_f D \frac{d^2h}{dx^2} = \frac{k_b (h_x - t_x)}{Z_b + \frac{t_x}{\overline{K}}} \qquad (112)$$

where the effective thickness of the seepage berm in regards to seepage is

$$t_x \frac{k_b}{k_t} = \frac{t_x}{\overline{K}} \qquad (113)$$

Therefore,

$$\frac{d^2h}{dx^2} = \frac{k_b}{k_f D Z_b} \left(\frac{h_x - t_x}{1 + \frac{t_x}{Z_b \overline{K}}} \right) \qquad (114)$$

35. It is assumed that the uplift safety factor for the top blanket and the seepage berm combined is a constant F . Then

$$F = \frac{Z_b\gamma_b' + t_x\gamma_t'}{(h_x - t_x)\ \gamma_w} \tag{115}$$

and

$$h_x - t_x = \frac{Z_b\gamma_b' + t_x\gamma_t'}{\gamma_w F} \tag{116}$$

$$h_x - \frac{Z_b\gamma_b'}{\gamma_w F} = t_x\left(1 + \frac{\gamma_t'}{\gamma_w F}\right) \tag{117}$$

$$t_x = \frac{h_x - \dfrac{Z_b\gamma_b'}{\gamma_w F}}{1 + \dfrac{\gamma_t'}{\gamma_w F}} \tag{118}$$

Equation 114 can be rewritten as

$$\frac{d^2h}{dx^2} = \frac{c^2\left(h_x + \dfrac{Z_b\gamma_b'}{\gamma_t'}\right)}{\dfrac{\gamma_w F h_x}{\gamma_t' Z_b \overline{K}} + \dfrac{\gamma_w F}{\gamma_t'} + 1 - \dfrac{\gamma_b'}{\gamma_t'\overline{K}}} \tag{119}$$

The author was not able to solve Equation 119, and so he resorted to a numerical method (finite difference) to solve it.

36. If $\overline{K}$ approaches infinity, then Equation 119 becomes Equation 19. If $\overline{K} = 1$ and $\gamma_b'/\gamma_t' = 1.0$, then Equation 119 becomes Equation 52. If $\overline{K} = 1.0$ and $\gamma_b'/\gamma_t' \neq 1.0$, then Equation 119 becomes Equation 67.

37. To convert Equation 119 for the numerical method, approximate

$$\frac{d^2h}{dx^2} \approx \frac{h_3 - 2h_2 + h_1}{\Delta x^2} \tag{120}$$

where

h_1 = uplift head at the base of the semipervious top blanket at $x = (n - 1)\Delta x$ (Figure 2)

h_2 = uplift head at the base of the semipervious top blanket at $x = n\Delta x$

h_3 = uplift head at the base of the semipervious top blanket at $x = (n + 1)\Delta x$

Δx = increment of horizontal distance

38. In general, h_1 and h_2 are known for a given safety factor, etc. Then h_3 can be solved by using the following expression

$$h_3 = 2h_2 - h_1 + \left[\frac{c^2\Delta x^2\left(h_2 + \frac{Z_b\gamma_b'}{\gamma_t'}\right)}{\frac{\gamma_w F h_2}{\gamma_t' Z_b \overline{K}} + \frac{\gamma_w F}{\gamma_t'} + 1 - \frac{\gamma_b'}{\gamma_t'\overline{K}}}\right] \tag{121}$$

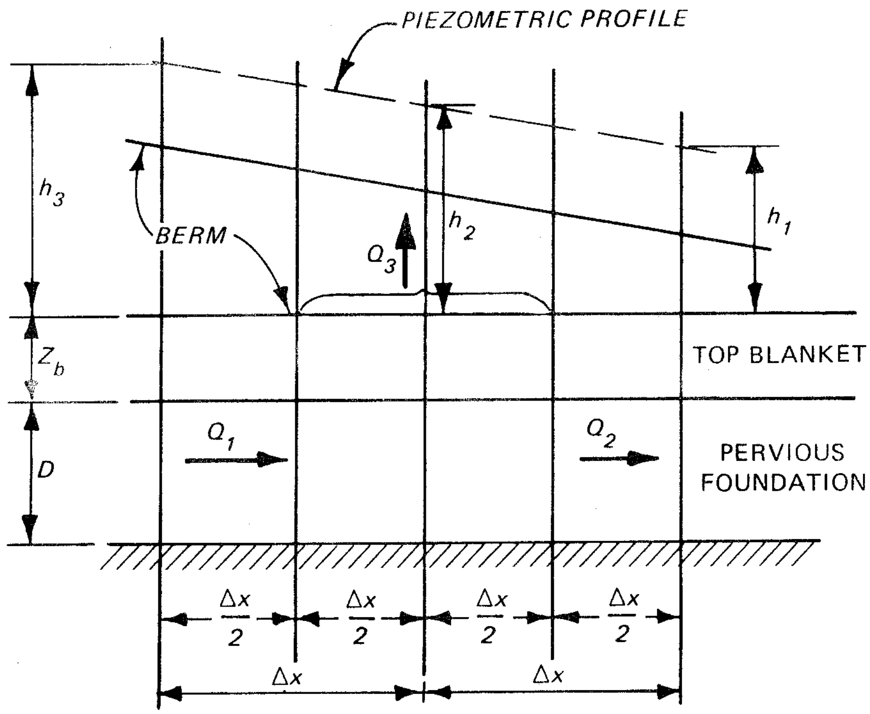
39. From Equation 2, the uplift head at the base of the top blanket at the landside seepage berm toe is

$$h_a = \frac{\gamma_b' Z_b}{\gamma_w F}$$

The gradient landward at this location is h_a/L_{LS} . These are boundary conditions that must be considered in applying the numerical method of solution. As shown in Figure 3, the initial value of h_1 is h_a . The initial value of h_2 is

$$h_{2_i} = \frac{Z_b\gamma_b'}{\gamma_w F}\left(\frac{\Delta x}{L_{LS}} + 1\right) + \frac{c^2\Delta x^2 Z_b\gamma_b'}{2\gamma_w F} \tag{122}$$

The initial value of h_{3_i} may then be computed using h_{a_1} and h_{2_i} and Equation 121. This procedure is used to obtain the next value of h_3 with h_{2_i} becoming h_1 and h_{3_i} becoming h_2 . The procedure is repeated across the base of the seepage berm for each increment, Δx . At the landside levee toe, another boundary condition must be met. This condition, the seepage gradient in the pervious foundation, is expressed as



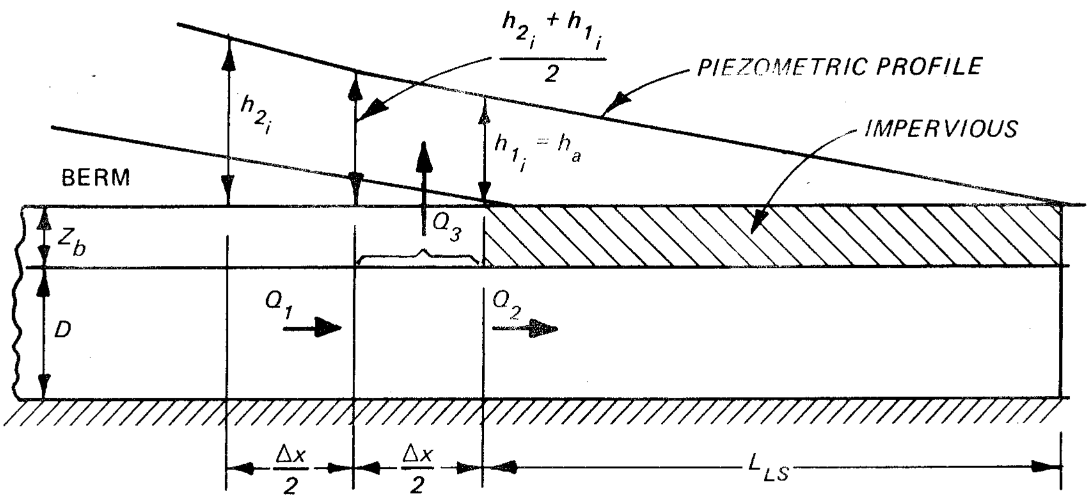
$$Q_1 - Q_2 - Q_3 = 0$$

$$Q_1 = \frac{k_f D(h_3 - h_2)}{\Delta x}, \quad Q_2 = \frac{k_f D(h_2 - h_1)}{\Delta x}$$

$$Q_3 = \frac{\frac{k_b}{Z_b}\left(h_2 + \frac{Z_b \gamma_b'}{\gamma_t'}\right)\Delta x}{\frac{\gamma_w F h_2}{\gamma_t' Z_b \overline{\overline{K}}} + \frac{\gamma_w F}{\gamma_t'} + 1 - \frac{\gamma_b'}{\gamma_t' \overline{\overline{K}}}}$$

$$h_3 = 2h_2 - h_1 + \frac{c^2 \Delta x^2 \left(h_2 + \frac{Z_b \gamma_b'}{\gamma_t'}\right)}{\frac{\gamma_w F h_2}{\gamma_t' Z_b \overline{\overline{K}}} + \frac{\gamma_w F}{\gamma_t'} + 1 - \frac{\gamma_b'}{\gamma_t' \overline{\overline{K}}}}$$

Figure 2. Derivation of uplift head at the base of the semipervious top blanket at $x = (n + 1)\Delta x$



$$Q_1 - Q_2 - Q_3 = 0$$

$$Q_1 = \frac{k_f D(h_{2_i} - h_a)}{\Delta x}, \quad Q_2 = \frac{k_f D h_a}{L_{LS}}$$

$$Q_3 = \frac{\frac{k_b}{Z_b}\frac{\Delta x}{2}\left(h_a + \frac{Z_b \gamma_b'}{\gamma_t'}\right)}{\frac{\gamma_w F h_a}{\gamma_t' Z_b K} + \frac{\gamma_w F}{\gamma_t'} + 1 - \frac{\gamma_b'}{\gamma_t' K}}$$

$$h_{2_i} = h_a\left(\frac{\Delta x}{L_{LS}} + 1\right) + \frac{c^2 \Delta x^2}{2} \frac{h_a + \frac{Z_b \gamma_b'}{\gamma_t'}}{\frac{\gamma_w F h_a}{\gamma_t' Z_b \bar{K}} + \frac{\gamma_w F}{\gamma_t'} + 1 - \frac{\gamma_b'}{\gamma_t' \bar{K}}}$$

But $h_a = \frac{Z_b \gamma_b'}{\gamma_w F} = h_{1_i}$

and so

$$h_{2_i} = \frac{Z_b \gamma_b'}{F \gamma_w}\left(\frac{\Delta x}{L_{LS}} + 1\right) + \left(\frac{c^2 \Delta x^2 Z_b \gamma_b'}{2\gamma_w F}\right) = \frac{Z_b \gamma_b'}{\gamma_w F}\left(\frac{\Delta x}{L_{LS}} + 1 + \frac{c^2 \Delta x^2}{2}\right)$$

Figure 3. Derivation of initial value of uplift head at the base of the semipervious top blanket near berm toe

$$i = \frac{H - h_t}{\overline{X} + L_2} \tag{123}$$

where

H = head on the system

h_t = uplift head at the base of the top blanket at the landside levee toe

$\overline{X}$ = effective length of the riverside blanket

L_2 = levee base width

40. The value of H is computed as shown in Figure 4 for each new value of h_3 using the following expression

$$H = h_3 \left(1 + \frac{\overline{X} + L_2}{\Delta x}\right) - h_2 \left(\frac{\overline{X} + L_2}{\Delta x}\right) + (\overline{X} + L_2) \frac{c^2 \Delta x}{2} \left(\frac{h_3 + \frac{Z_b \gamma_b'}{\gamma_t'}}{\frac{\gamma_w F h_3}{\gamma_t' Z_b \overline{K}} + \frac{\gamma_w F}{\gamma_t'} + 1 - \frac{\gamma_b'}{\gamma_t' \overline{K}}} \right) \tag{124}$$

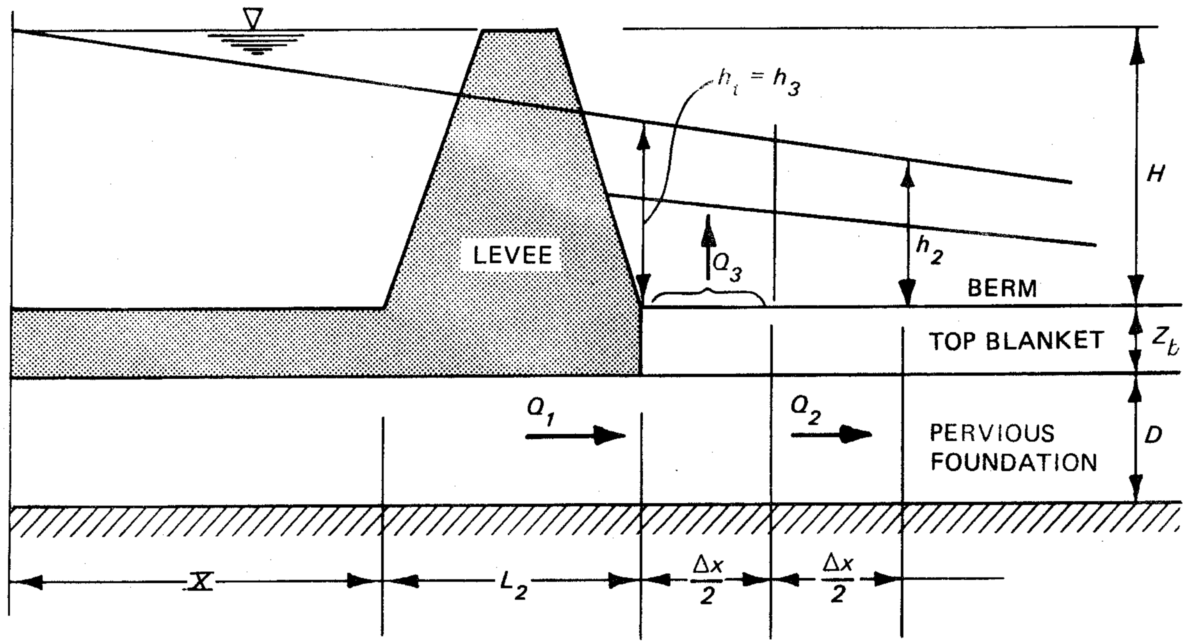
A calculated value of H that is generally less than the given value of H indicates that the number of increments, $n\Delta x$, used is not equal to B, the seepage berm width. After a number of steps, a calculated value of H that exceeds the given value of H indicates that the sum of $n\Delta x$ is larger than B. The values of B and h_t are then found by interpolation.

41. The thickness of the seepage berm, t_x, is calculated for Δx using h_3.

$$t_x = \frac{h_3 \gamma_w F - Z_b \gamma_b'}{\gamma_w F + \gamma_t'} \tag{125}$$

Using the data as given in the examples presented in Case I (see paragraph 5) with $\overline{K} = 1.0$, $F = 1.0$, and $\Delta x = 10$ ft, then

$$h_a = \frac{1 \times 5}{1} = 5 = h_{1_i}$$



$$Q_1 - Q_2 - Q_3 = 0$$

$$Q_1 = \frac{k_f D(H - h_t)}{(\bar{X} + L_2)}, \quad Q_2 = \frac{k_f D(h_t - h_2)}{\Delta x}$$

$$Q_3 = \frac{\dfrac{\Delta x}{2}\dfrac{k_b}{Z_b}\left(h_t + \dfrac{Z_b \gamma'_b}{\gamma'_t}\right)}{\dfrac{\gamma_w F h_t}{\gamma'_t Z_b \bar{K}} + \dfrac{\gamma_w F}{\gamma'_t} + 1 - \dfrac{\gamma'_b}{\gamma'_t \bar{K}}}$$

$$H = h_t\left(1 + \frac{\bar{X} + L_2}{\Delta x}\right) - h_2\left(\frac{\bar{X} + L_2}{\Delta x}\right)$$

$$+ \frac{\left(\dfrac{\bar{X} + L_2}{\Delta x}\right)\dfrac{\Delta x^2 c^2}{2}\left(h_t + \dfrac{Z_b \gamma'_b}{\gamma'_t}\right)}{\dfrac{\gamma_w F h_t}{\gamma'_t Z_b \bar{\bar{K}}} + \dfrac{\gamma_w F}{\gamma'_t} + 1 - \dfrac{\gamma'_b}{\gamma'_t \bar{\bar{K}}}}$$

Figure 4. Derivation of head on system for semipervious berm

$$h_{2_i} = 5\left(\frac{10}{223.6} + 1\right) + \frac{100 \times 5}{2 \times 50{,}000} = 5.228623596$$

$$h_3 = 2h_2 - h_1 + \frac{\frac{100}{50{,}000}(h_2 + 5)}{\frac{h_2}{5} + 1} = 2h_2 - h_1 + 0.01$$

$$H = h_3\left(1 + \frac{218.5 + 200}{10}\right) - h_2\left(\frac{218.5 + 200}{10}\right)$$

$$+ \frac{\frac{218.5 + 200}{10} \times \frac{100}{2 \times 50{,}000}(h_3 + 5)}{\frac{1 \times 1 \times h_3}{5 \times 1} + (1 \times 1) + 1 - (1 \times 1)} = 42.85\ h_3 - 41.85 + 0.20925$$

Values of t_x , H , and B_x are given in the following tabulation.

N	h_3 , ft	t_x , ft	Calculated H , ft	B_x , ft
-1	5.0*	0.00	--	0
0	5.229**	0.11	--	10
1	5.47	0.23	15.66	20
2	5.72	0.36	16.33	30
3	5.97	0.49	17.01	40
4	6.24	0.62	17.69	50
5	6.52	0.76	18.39	60
6	6.81	0.91	19.10	70
7	7.11	1.05	19.82	80
8	7.42	1.21	20.54	90
9	7.74	1.37	21.28	100
10	8.06	1.53	22.03	110
11	8.40	1.70	22.78	120
12	8.75	1.88	23.55	130
13	9.11	2.06	24.33	140
14	9.48	2.24	25.12	150
15	9.86	2.43	25.91	160
16	10.25	2.62	26.72	170
17	10.65	2.82	27.54	180
18	11.05	3.03	28.36	190
19	11.47	3.24	29.20	200
20	11.96	3.45	30.05	210

* $5 = h_{1_i}$

** $5.229 = h_{2_i}$

42. The last increment in the tabulation above is too large; the corrected increment, δ , is $\frac{30 - 29.20}{30.05 - 29.20} \times 10 = 9.4$ ft . The berm width, B , is $200 + 9.4 = 209.4$ ft . The seepage berm thickness, t , at the landside levee toe for B is

$$\frac{9.4}{10} = \frac{X - 3.24}{3.45 - 3.24}$$

$$0.94 \times 0.21 + 3.24 = 3.44 \text{ ft}$$

It should be noted that B_x is 20 ft for the initial h_3 , and

$$t_x = \frac{h_3 - 5}{1 + 1} = \frac{h_3}{2} - 2.5$$

43. Also, note that for $\overline{K}$ equal to or larger than one, the theoretical berm thickness at the landside berm toe is zero. For $\overline{K}$ less than one, then the berm thickness at the landside toe is finite.

44. For $\overline{K}$ equal to 10 or larger, the berm length is nearly equal to that for $\overline{K}$ equal to infinity. The need to solve Equation 121 is academic and is not required for practical design.

45. The factor of safety, F , in Equation 105 has been assumed to be a constant. However, a safety factor can be used that is a function of x . A simple linear variation of the safety factor is

$$F = F_B + (F_o - F_B)\frac{x}{B} \tag{126}$$

where

F_o = uplift safety factor at the landside seepage berm toe (x = 0)

F_B = uplift safety factor at the landside levee toe (x = B)

B = length of the seepage berm

The difference in Equations 126 and 85 is caused by the different location of the x origin.

46. Equation 126 can be inserted into Equation 121. The procedure is to estimate B and then to proceed in a step-by-step manner

using Δx as selected. At x equal to B, a determination of H, the net head acting upon the system, is made. If the computed H is not equal to the actual H, then a new estimate of B is made and a new value of H determined. In general, this new value of H will not be equal to the actual value of H, so a new estimate of B is made and the procedure reiterated. Usually, two computed values of H will permit a close estimation of H and, thus, the needed value of B. If required, a plot of B versus H can be made, and the value of B for the actual value of H can be determined. Results of such a study are shown in Figure 5, where $F_o = 1.0$ and $F_B = 1.5$ for the condition that the permeabilities of the seepage berm are equal to or larger than that of the top blanket, i.e. $\overline{K} \geq 1$.

Case VIII - General Case

47. The previously presented cases have either a constant uplift safety factor or one that varies in a linear manner. For this case, it is assumed that the berm thickness is a function of x. The uplift safety factor is dependent upon the assumed berm thickness and, in general, it will not be a constant. The basic differential equation is based upon Equation 114.

$$\frac{d^2h}{dx^2} = \frac{k_b}{k_f D Z_b}\left(\frac{h_x - t_x}{1 + \frac{t_x k_b}{Z_b k_t}}\right) = c^2 \left[\frac{h_x - f(x)}{1 + \frac{f(x)}{Z_b \overline{K}}}\right] \qquad (127)$$

48. No attempt has been made to solve Equation 125 in a formal manner; rather, the numerical method, as presented in Case VII, is used. The uplift head, h_x, at the base of the semipervious top blanket is computed in steps. The uplift safety factor is also computed for each step. If k_t is less than k_b, the value of F is found using the following expression, which applies only to the seepage berm. The seepage gradient up through the effective top blanket is (using Equation 65)

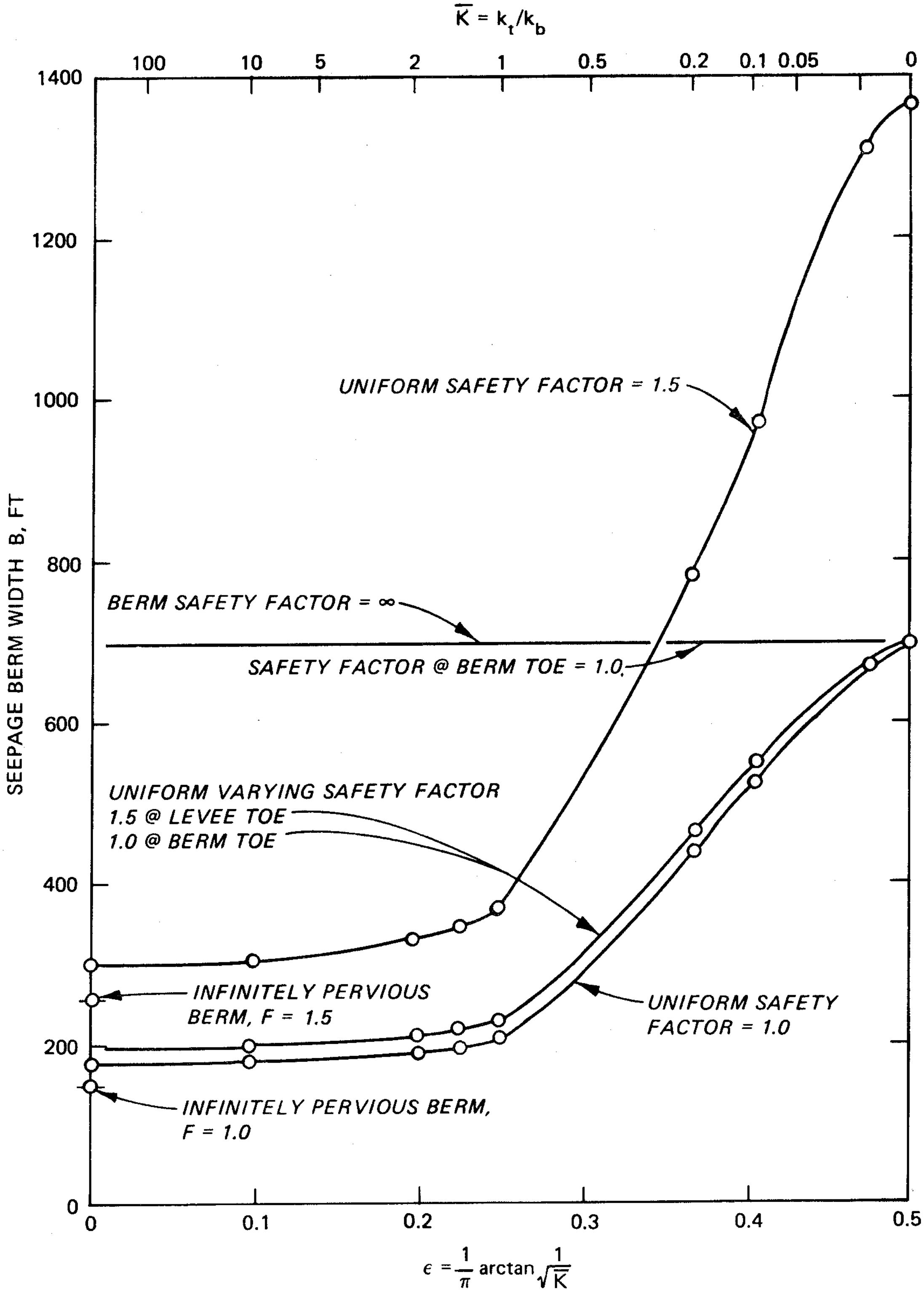


Figure 5. Relationship between seepage berm width and ratio of permeability of seepage berm to top blanket for various safety factors

$$i = \frac{h_x - t_x}{t_x + \frac{Z_b k_t}{k_b}} = \frac{h_x - t_x}{t_x + Z_b \overline{K}}$$

The safety factor against seepage uplift is

$$F = \frac{\gamma_t'}{\gamma_w i} = \frac{\gamma_t'}{\gamma_w} \left(\frac{t_x + Z_b \overline{K}}{h_x - t_x} \right) \tag{128}$$

If k_t is greater than k_b , then

$$F = \frac{\gamma_t'}{\gamma_w} \left(\frac{t_x + \frac{Z_b \gamma_b'}{\gamma_t'}}{h_x - t_x} \right) \tag{129}$$

If the uplift head at the base of the top blanket extends just to the upper surface of the seepage berm, then the safety factor is infinite. For cases in which the uplift head is below the upper surface of the seepage berm, the uplift safety factor is infinite and the basic differential Equation 127 is no longer valid. The seepage berm then becomes uneconomical; its thickness is excessive. In fact, to obtain economy, the uplift head at the base of the top blanket should always be above the upper surface of the seepage berm except for the exceptional case when the berm is constructed of very pervious soil.

Conclusions

49. A plot of the variation of the seepage berm width, B , various values of the berm top blanket seepage ratio, $\overline{K}$, and safety factors, F , is presented in Figure 5. It should be noted that B is very sensitive to $\overline{K}$ for values of $\overline{K}$ less than 1.0 but not sensitive to $\overline{K}$ for values of $\overline{K}$ greater than 1.0. The curves in Figure 5 show the condition that the uplift safety factor varies in a uniform manner. For instance, if the uplift safety factor varies from 1.5 at the landside levee toe to 1.0 at the landside seepage berm toe, then the berm width is only slightly greater than that for a uniform safety factor

of 1.0. Of course, near the levee the berm thickness is greater for the case where the uplift safety factor varies than that for a uniform condition of the uplift safety factor. Also shown is a curve where the uplift safety factor is infinite under the seepage berm but 1.0 at the landside berm toe. This condition exists when the piezometric profile for the head at the base of the top blanket coincides with the upper surface of the seepage berm. The berm width is the same as that for an impervious berm.

50. It should be noted that if the uplift safety factor is greater than 1.0 at the berm toe, then as the top blanket becomes less thick, the berm width becomes longer and for thin top blankets the berm width becomes extremely long. Thus, for thin top blankets, other means of seepage control should be investigated.

51. The uplift safety factor near the landside levee toe should be such that the horizontal stresses in the levee (earth pressures and seepage forces) can be transferred into the landside foundation. This condition may require the uplift safety factor to vary from a higher value at the levee toe to a lesser value at the berm toe. No criteria can be given for this condition, but they will be dependent upon stability studies.

52. When the seepage berm is impervious, the berm width is a maximum. When the seepage berm is infinitely pervious, then the berm width is a minimum. Thus, the more seepage permitted up through the seepage berm, the shorter the berm may be. Hence, seepage berms should be made of the most pervious soils available in the interest of economy.

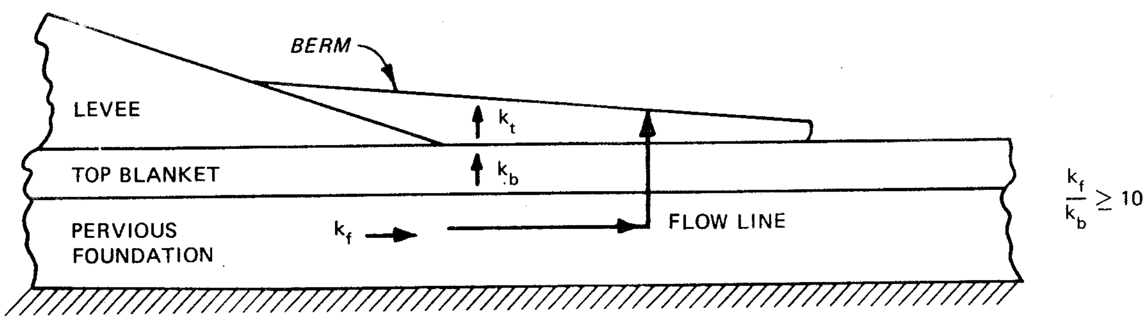
53. Most of the cases studied in this report have a seepage berm that is concaved upward. It may appear that such a berm could be more difficult to construct, so a berm having a uniform slope from the levee to the berm toe should be used. If both berms have common points at the levee and berm toe, then the uniform sloping berm will contain more material than the concave berm. Thus, the seepage up through the uniform sloping berm will be less than that for the concave berm. Also, the uplift under the berm and the top blanket will be greater and the safety factors will be less than those for the concave berm; hence,

a concave berm is the most effective and economical.

54. Because of the great difficulty in determining the permeability of the foundation, the semipervious top blanket, and the seepage berm, the equations presented should be used only as a guide to good judgement. In all studies, a range of permeability values should be used and not average values. In all cases, the uplift safety factor at the landside levee toe should be greater than 1.0. However, the uplift safety factor at the landside seepage berm toe may be 1.0 or greater.

55. The design of a seepage berm using the criteria of the seepage safety factor greater than 1.0 can result in berms of excessive lengths. For practical cases, it may be better to use a berm with a proper uplift safety factor at the landside levee toe and a length, B, such that the uplift safety factor at the landside berm toe is 1.0 or less. This could result in landside seepage boils, but the possibility of these boils endangering the levee would be minimal provided the berm length is a reasonable value. The design of the berm should consider the pipeability of the foundation and top blanket soils. Furthermore, the berm design is probably more dependent upon sound engineering judgement than upon mathematical theory.

56. A summary of the cases and the applicable permeability coefficients are given in Figure 6.



CASE	DESCRIPTION	BERM			TOP BLANKET			COMPARISON OF BERM & TOP BLANKET PERMEABILITY	FIELD BERM
		k_t	k_{t_H}	k_{t_v}	k_b	k_{b_H}	k_{b_v}		
I	IMPERVIOUS BERM	0	0	0				$k_b >>> k_t$	CLAY
II	INFINITELY PERVIOUS BERM	∞	∞	∞				$k_t >>> k_b$	SAND WITH DRAIN
III	INFINITELY PERVIOUS BERM	—	0	∞				$k_{t_v} >>> k_{b_v}$	SAND
IV	BERM PERVIOUSNESS EQUAL THAT TOP BLANKET	—	0	k_{b_v}	—	0	k_{t_v}	$k_{b_v} = k_{t_v}$	SAND OR SILT
V	SEMIPERVIOUS BERM		0		—	0		$k_{b_v} \geq k_{t_v}$	SAND OR SILT
VI	VARIABLE SAFETY FACTOR		0			0		$k_{b_v} \geq k_{t_v}$	SAND OR SILT
VII	PERVIOUS BERM		0			0		$k_{t_v} \geq k_{b_v}$	SAND
VIII	GENERAL CASE		0			0		$k_{t_v} \gtrless k_{B_v}$	

Figure 6. Summary of relationships between seepage berm and top blanket permeabilities investigated

Appendix A: Notation

$\overline{A}$	Constant
B	Width of the seepage berm from the landside levee toe to the landside berm toe
$\overline{B}$	Constant
c	$(k_b/k_f DZ_b)^{1/2}$
C_1	Constant
C_2	Constant
$\overline{C}$	Constant
D	Thickness of the pervious foundation
$\overline{D}$	Constant
F	Uplift safety factor of the top blanket or the seepage berm (F_t) or the combination of both (F_{t+b})
F_o	Uplift safety factor at the landside levee toe
F_x	Safety factor at point x
F_B	Uplift safety factor at the landside berm toe
h_a	Allowable seepage uplift head at the landside seepage berm toe (The head is at the base of the top blanket but measured upward from the upper surface of the top blanket.)
h_t	Seepage uplift head at the base of the top blanket under the landside levee toe
h_x	Seepage uplift head at point x from the coordinate origin. The head is at the base of the top blanket but measured upward from the upper surface of the top blanket
h_1, h_2, h_3	Uplift head at the base of the semipervious top blanket at $x = (n - 1)\Delta x$, $x = n\Delta x$, and $(n + 1)\Delta x$, respectively
H	Difference in the hydraulic head between the river flood level and the landside upper surface of the natural top blanket (or landside pool, if such exists)
i	Seepage gradient

k_b Vertical permeability coefficient of the top blanket

k_f Horizontal permeability coefficient of the pervious foundation

k_t Vertical permeability coefficient of the seepage berm

$\overline{K}$ Ratio of the seepage berm permeability to that of the top blanket = k_t/k_b

L_1 Length of the riverside top blanket measured from the riverbank to the riverside levee toe

L_2 Base width of the levee

L_{LS} Effective length of the landside top blanket measured landward from the landside seepage berm toe (If the length of the landside top blanket is infinite, then $L_{LS} = 1/c$. For Case II, L_{LS} is measured from the landside levee toe.)

t Maximum thickness of the seepage berm located at the landside levee toe

t_x Thickness of the seepage berm at point x

x Horizontal distance measured landward from downstream toe of the levee or dam

$\underline{\overline{X}}$ Effective length of the riverside top blanket;

$$\underline{X} = \frac{\tanh\ (cL_1)}{c}$$

Z_b Thickness of the natural top blanket

γ_b' Buoyant unit weight of the top blanket

γ_m' Moist unit weight of the seepage berm

γ_t' Buoyant unit weight of the seepage berm

γ_w Unit weight of water

Δx Increment of horizontal distance

ε $\frac{1}{\pi}$ arc tan $\frac{1}{\sqrt{\overline{K}}}$

θ Constant

ϕ Variable

Supplement No. 1

SEEPAGE BERM WITH CONSTANT SLOPE OF UPPER SURFACE

<u>Contents</u>

MATHEMATICAL ANALYSES OF LANDSIDE SEEPAGE BERMS

SEEPAGE BERM WITH CONSTANT SLOPE OF UPPER SURFACE

Introduction

1. This report is Supplement No. 1 to Technical Report REMR-GT-1. The mathematical solution for the case of a landside seepage berm with a constant slope of its upper surface is presented. Examples are included in this supplement to illustrate the procedure.

Assumptions

2. The assumptions are those of the main report.

Case IX - Berm with Constant Outer Slope

General solution A

3. This is a special case of Case VIII - General Case presented in Technical Report REMR-GT-1, "Mathematical Analyses of Landside Seepage Berms," hereafter referred to as the main report. The coordinate origin is located at the landside berm toe and the horizontal distance measured landward from the downstream toe of the levee or dam x * is positive to riverward. A typical section of the geotechnical conditions is given in Figure 1 of this supplement. The riverside and landside semi-pervious top blankets are transformed to effective lengths, $\underline{\bar{X}}$ and L_{LS} , respectively. The top blankets of the transformed section are assumed to be impervious. The length of top blanket beneath the berm is not transformed. The landside effective length, L_{LS} , is

$$L_{LS} = \left(\frac{k_f Z_b D}{k_b}\right)^{1/2} = \frac{1}{c} \tag{1}$$

and the riverside effective length, $\underline{\bar{X}}$, is

$$\underline{\bar{X}} = \frac{\tanh\ (cL_1)}{c} \tag{2}$$

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* Symbols are listed and defined in the Notation (Appendix A).

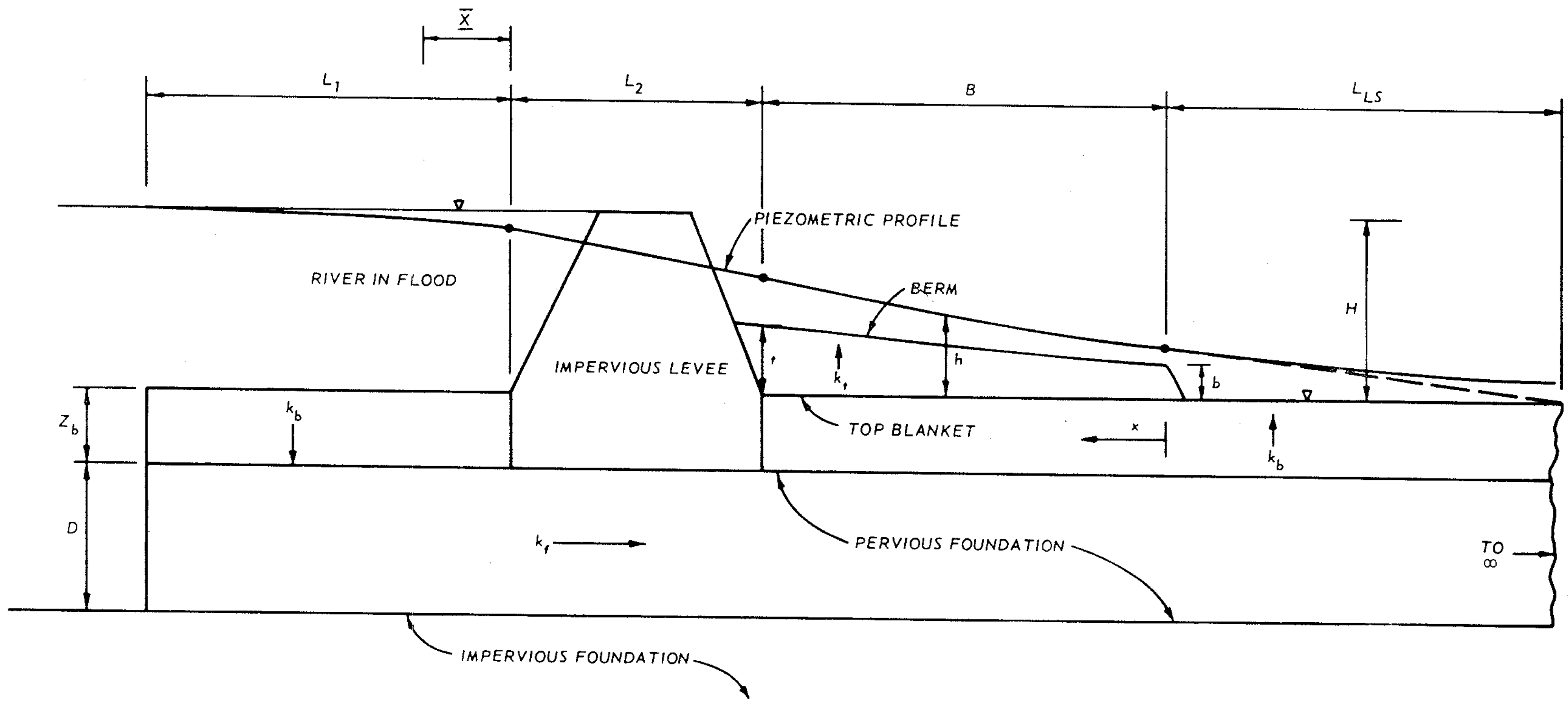

Figure 1. Generalized cross section of geologic strata, leveed seepage berm

where

$$c = \left(\frac{k_b}{k_f Z_b D}\right)^{1/2} \tag{3}$$

and

k_f = the horizontal coefficient of permeability of the pervious foundation

Z_b = the semipervious top blanket's thickness

D = the pervious foundation thickness

k_b = the vertical coefficient of permeability of the semipervious top blanket

L_1 = the distance from the riverside levee toe to the riverbank

The cross section of the transformed conditions is shown in Figure 2.

4. The thickness of the seepage berm t_x at a distance x from the landside berm toe is

$$t_x = \frac{x(t - b)}{B} + b \tag{4}$$

where

t = the thickness of the berm at the landside levee toe ($x = B$)

b = the thickness of the berm at the landside toe ($x = 0$)

B = the width of the seepage berm (the distance from the landside berm toe to the landside levee toe)

5. The seepage uplift safety factor (hereafter referred to as the safety factor) will vary from F_o at the berm toe to F_B at the landside levee toe. The variation of the safety factor will be dependent upon the geotechnical conditions and net head at the site. The safety factor F_o for the top blanket (at $x = 0$) is

$$F_o = \left(\frac{Z_b \gamma_b'}{h_a \gamma_w}\right) \tag{5}$$

where

γ_b' = the buoyant unit weight of the top blanket

h_a = the allowable seepage uplift head at the base of the top blanket, at the berm toe, measured upward from the top of the landside tailwater considered to be the upper surface of the top blanket

γ_w = the unit weight of water

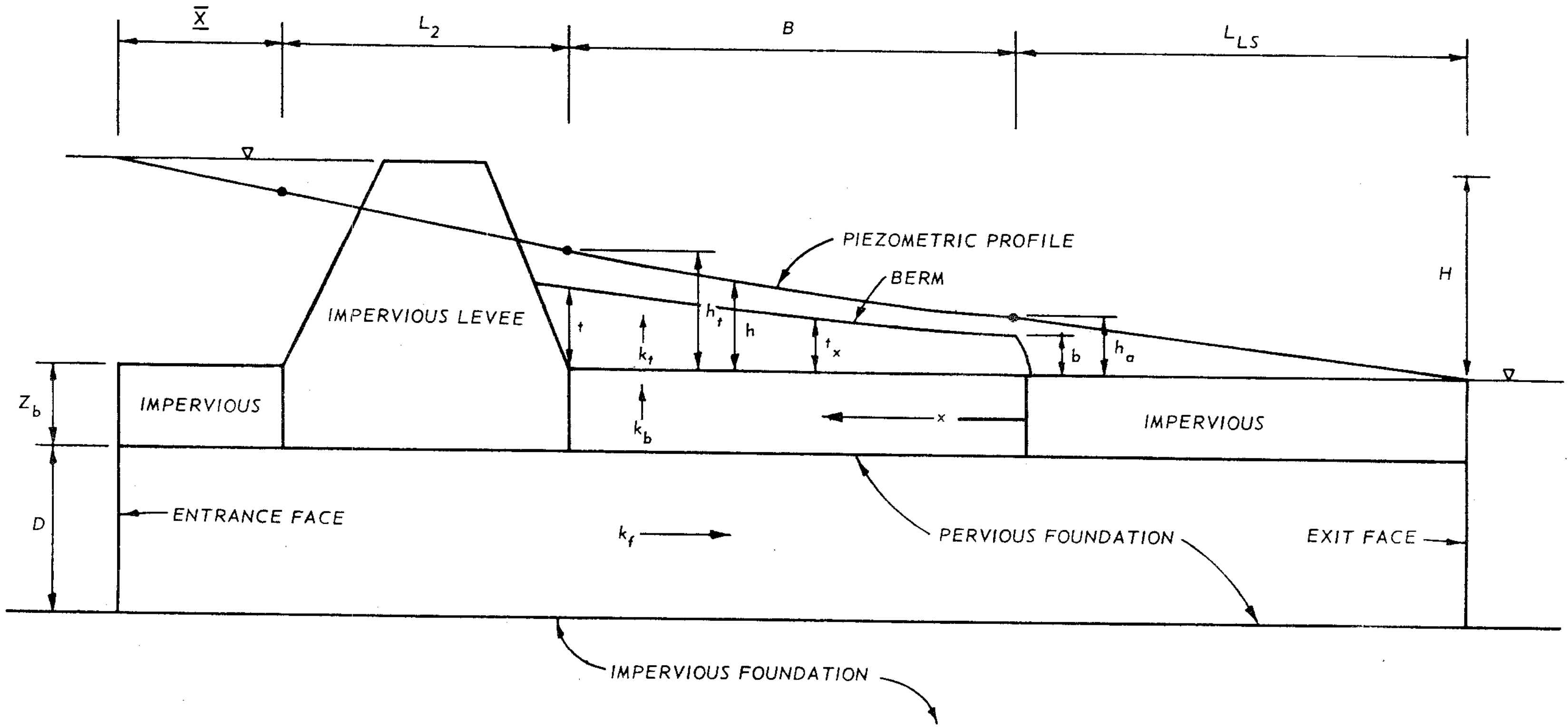

Figure 2. Transformed cross section

Rearranging Equation 5 gives

$$h_a = \frac{Z_b \gamma_b'}{F_o \gamma_w} \tag{6}$$

The need for a landside berm toe of thickness b results from the assumption that the horizontal permeabilities of both the berm and the top blanket are zero. As noted in paragraph 24 (pp 27-28) of the main report, the top blanket thickness Z_b may be transformed to an effective thickness Z_b' so that seepage up through the top blanket will have the same permeability as the berm k_t :

$$Z_b' = \frac{Z_b k_t}{k_b} = Z_b \overline{K} \tag{7}$$

where the permeability ratio is

$$\overline{K} = \frac{k_t}{k_b} \tag{8}$$

The upward seepage gradient i in the berm toe is

$$i = \frac{h_a - b}{b + Z_b'} = \frac{h_a - b}{b + Z_b \overline{K}} \tag{9}$$

The seepage uplift safety factor F_o at the berm toe for the berm is

$$F_o = \frac{(b + Z_b \overline{K}) \gamma_t'}{(h_a - b) \gamma_w} = \frac{(b + Z_b \overline{K}) \gamma_t'}{\left(\frac{Z_b \gamma_b'}{\gamma_w F_o} - b \right) \gamma_w} \tag{10}$$

where γ_t' equals the buoyant unit weight of berm. Rearranging Equation 10 gives

$$b = \frac{Z_b \left(\frac{\gamma_b'}{\gamma_t'} - \overline{K} \right)}{\left(1 + \frac{\gamma_w F_o}{\gamma_t'} \right)} \tag{11}$$

When the ratio of unit weight γ_b'/γ_t' is equal to or less than $\bar{K}$, then the berm toe thickness b is zero.

6. The second order differential equation relating the change in the pervious foundation seepage with the upward seepage through the top blanket and the berm is

$$k_f D \frac{d^2h}{dx^2} = \frac{k_t(h - t_x)}{(Z_b\bar{K} + t_x)} \tag{12}$$

where h equals the seepage uplift head (at point x) at the base of the top blanket but measured upward from the upper surface of the top blanket. Dividing both sides of Equation 12 by $k_f D$ gives

$$\frac{d^2h}{dx^2} = \frac{k_t(h - t_x)}{k_f D(Z_b\bar{K} + t_x)} \frac{k_b}{k_b} = \frac{\bar{K}c^2(h - t_x)}{\bar{K} + \frac{t_x}{Z_b}} \tag{13}$$

Now set

$$h - t_x = h - (t - b)\frac{x}{B} - b = y \tag{14}$$

and differentiate y in respect to x :

$$\frac{dh}{dx} - \frac{t - b}{B} = \frac{dy}{dx} \tag{15}$$

and again:

$$\frac{d^2h}{dx^2} = \frac{d^2y}{dx^2} \tag{16}$$

Also set

$$\bar{K} + \frac{t_x}{Z_b} = \bar{K} + \frac{(t - b)x}{Z_b B} + \frac{b}{Z_b} = \zeta \tag{17}$$

where ζ equals a variable. Differentiate ζ with respect to x , which gives

$$\frac{d\zeta}{dx} = \frac{t - b}{Z_b B} \tag{18}$$

and

$$dx^2 = \left(\frac{Z_b B}{t - b}\right)^2 d\zeta^2 \tag{19}$$

Substituting Equations 14, 16, and 19 into Equation 13 produces

$$\frac{d^2 y}{d\zeta^2} = \left(\frac{Z_b B}{t - b}\right)^2 \overline{K} c^2 \frac{y}{\zeta} = \psi \frac{y}{\zeta} \tag{20}$$

where

$$\psi = \left(\frac{Z_b B c}{t - b}\right)^2 \overline{K} \tag{21}$$

7. The solution of Equation 20 is

$$y = h - t_x = 2\sqrt{\psi\zeta} \left[C_1 I_1(2\sqrt{\psi\zeta}) + C_2 K_1(2\sqrt{w\zeta})\right] \tag{22}$$

and

$$\frac{dy}{d\zeta} = 2\psi\left[C_1 I_o(2\sqrt{\psi\zeta}) - C_2 K_o(2\sqrt{\psi\zeta})\right] \tag{23}$$

The functions $I_o(\ )$, $I_1(\ )$, $K_o(\ )$, and $K_1(\ )$ are modified Bessel functions of the first and second kinds of order zero and one, respectively. Tabulations of these functions are included in H. B. Dwight.* The coefficients C_1 and C_2 are determined using the boundary conditions at the berm toe (x = 0):

$$h = h_a = \frac{Z_b \gamma_b'}{F_o \gamma_w} , \quad \frac{dh}{dx} = h_a c = \frac{h_a}{L_{LS}} , \quad y = h_a - b$$

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* H. B. Dwight. 1961. "Mathematical Tables," Dover Publishing Company, New York, pp 184-193. Bessel functions are also discussed in Appendix B of this supplement.

$$\frac{dy}{dx} = \frac{dh}{dx} - \left(\frac{t - b}{B}\right) = h_a c - \left(\frac{t - b}{B}\right), \quad \zeta_o = \bar{K} + \frac{b}{Z_b}$$

Now

$$\frac{dy}{dx}\frac{dx}{d\zeta} = \left(h_a c - \frac{t - b}{B}\right)\frac{Z_b B}{t - b} = \left(\frac{h_a Z_b Bc}{t - b} - Z_b\right) \tag{24}$$

Set

$$2\sqrt{\psi\zeta} = \omega_x \tag{25}$$

and

$$2\sqrt{\psi\zeta_o} = \omega_o \tag{26}$$

and also

$$2\sqrt{\psi\zeta_B} = \omega_B \tag{27}$$

where

ω_x = a variable

ω_B = a constant

ω_o = a constant

Then at the berm toe ($x = 0$), Equations 22 and 23 may be written as

$$y = h_a - b = \omega_o\left[C_1 I_1(\omega_o) + C_2 K_1(\omega_o)\right] \tag{28}$$

$$\frac{dy}{d\zeta} = Z_b\left(\frac{h_a Bc}{t - b} - 1\right) = 2\psi\left[C_1 I_o(\omega_o) - C_2 K_o(\omega_o)\right] \tag{29}$$

Solving the simultaneous Equations 28 and 29 and noting that

$$I_o(x)K_1(x) + I_1(x)K_o(x) = \frac{1}{x} \tag{30}$$

gives

$$C_1 = (h_a - b)\, K_o(\omega_o) + \left[\left(h_a - \frac{t - b}{Bc}\right)\sqrt{1 + \frac{b}{Z_b\overline{K}}}\,\right] K_1(\omega_o) \tag{31}$$

$$C_2 = (h_a - b)\, I_o(\omega_o) - \left[\left(h_a - \frac{t - b}{Bc}\right)\sqrt{1 + \frac{b}{Z_b\overline{K}}}\,\right] I_1(\omega_o) \tag{32}$$

8. At the landside levee toe, $x = B$, $h = h_t$, $y = h_t - t$,

$$\frac{dy}{d\zeta} = \frac{dy}{dx}\frac{dx}{d\zeta} = \left(\frac{dh}{dx} - \frac{t - b}{B}\right)\frac{Z_b B}{t - b}$$

and

$$\omega_B = \frac{2Z_b Bc\overline{K}}{t - b}\sqrt{1 + \frac{t}{Z_b\overline{K}}} \tag{33}$$

At the landside levee toe $(x = B)$

$$\frac{dh}{dx} = \frac{H - h_t}{\underline{X} + L_2} \tag{34}$$

Equations 22 and 23 become $(x = B)$

$$h_t = t + \omega_o\left[C_1 I_1(\omega_B) + C_2 K_1(\omega_B)\right] \tag{35}$$

and

$$\frac{dy}{d\zeta} = \left(\frac{H - h_t}{\underline{X} + L_2} - \frac{t - b}{B}\right)\frac{Z_b B}{t - b} = 2\psi\left[C_1 I_o(\omega_B) - C_2 K_o(\omega_B)\right] \tag{36}$$

After an algebraic operation, Equation 36 becomes

$$H = h_t + (\underline{X} + L_2)\left\{\frac{t - b}{B} + \frac{2Z_b Bc^2\overline{K}}{t - b}\left[C_1 I_o(\omega_B) - C_2 K_o(\omega_B)\right]\right\} \tag{37}$$

The value of ψ is given by Equation 21.

9. One must assume values of the berm width B and thickness t at the landside levee toe. Using Equation 35 and 37 values of H, the net head in the system is calculated. This calculated value of H will generally not agree with the design value. In addition the calculated safety factor F_B at the landside levee toe will generally not be equal to the design assumption. A systematic method is to assume a value of B and various values of t. Then calculate H and F_B. A curve is then drawn on a plot of H versus F_B for the selected B value and the value of F_B is found for H calculated equal to the design H. A curve of H versus t is also drawn and the value of t picked off for the design head H. The value of F_B will not be equal to the design value, so other values of B and t are selected and the procedure repeated. Then the values of F_B and t for the design head H are plotted against B and the values of B and t for the design value of H and F_B found. In some cases simple interpolation may be sufficient.

10. The safety factor at point x, F_x, is dependent upon the geotechnical and geometrical properties of the berm and foundation. The expression for the safety factor for the combined berm and top blanket is

$$F_x(\text{combined}) = \frac{Z_b\gamma_b' + t_x\gamma_t'}{(h - t_x)\gamma_w} \tag{38}$$

The safety factor for the berm only is

$$F_x(\text{berm}) = \frac{(Z_b\overline{K} + t_x)\ \gamma_t'}{(h - t_x)\ \gamma_w} \tag{39}$$

The lower safety factor should govern. As noted on page 35 of the main report, if

$$\overline{K} \gtrless \frac{\gamma_b'}{\gamma_t'}$$

then

$$F_x(\text{berm}) \gtrless F_x(\text{combined})$$

Equations 35 and 37 may be used to obtain the solutions for the limiting cases when $\overline{K} = 0$ and $\overline{K}$ approaches infinity. A more direct approach will be used and the basic differential equation (Equation 13) will be modified for these two limits.

<u>Solution B, $\overline{K} = 0$</u>

11. Equation 13 becomes

$$\frac{d^2h}{dx^2} = 0 \tag{40}$$

The solution is

$$h = C_3x + C_4 \tag{41}$$

At the berm toe, $x = 0$, $h = h_a = C_4$, and $dh/dx = h_ac = C_3$ so that

$$h = h_a(xc + 1) \tag{42}$$

At the landside levee toe, $x = B$, $h = h_t$ and $dh/dx = (H - h_t)/(\underline{X} + L_2)$. Using Equation 42

$$h_t = h_a(Bc + 1) \tag{43}$$

$$\frac{dh}{dx} = h_ac = \frac{H - h_t}{\underline{X} + L_2} = \frac{H - h_a(Bc + 1)}{\underline{X} + L_2} \tag{44}$$

and so

$$B = \frac{H}{h_ac} - \frac{1}{c} - \underline{X} - L_2 = \frac{HL_{LS}\gamma_wF_o}{Z_b\gamma_b'} - (\underline{X} + L_2 + L_{LS}) \tag{45}$$

which is also Equation 5 of the main report. No further discussion of this case will be given.

Solution C, $\overline{K}$ approaches infinity

12. It is considered that the horizontal permeability of the berm is zero. Thus the berm produces a head equal to t_x , which is a back pressure which reduces the upward seepage through the top blanket. Equation 13 becomes

$$\frac{d^2h}{dx^2} = \frac{k_b(h - t_x)}{k_f D Z_b} = c^2(h - t_x) \tag{46}$$

Because the berm thickness b at the land toe of the berm is zero, Equation 4 becomes

$$t_x = \frac{xt}{B} \tag{47}$$

Set

$$h - t_x = h - \frac{xt}{B} = y \tag{48}$$

so that Equation 46 becomes

$$\frac{d^2y}{dx^2} = c^2y \tag{49}$$

The solution is

$$y = h - t_x = h - \frac{xt}{B} = C_5e^{cx} + C_6e^{-cx} \tag{50}$$

At $x = 0$ (berm toe), $h = h_a$ and $dh/dx = h_a c$. Therefore,

$$h_a = C_5 + C_6 \tag{51}$$

and

$$\frac{dh}{dx} - \frac{t}{B} = \frac{dy}{dx} = c\ C_5 - c\ C_6 \tag{52}$$

so that

$$h_a - \frac{t}{Bc} = C_5 - C_6 \tag{53}$$

Solving for C_5 and C_6 results in

$$C_5 = h_a - \frac{t}{2Bc} \tag{54}$$

$$C_6 = \frac{t}{2Bc} \tag{55}$$

Thus, Equation 50 may be written as

$$h = \frac{xt}{B} + h_a e^{cx} - \frac{t}{2Bc} e^{cx} + \frac{t}{2Bc} e^{-cx} = \frac{xt}{B} + h_a e^{cx} - \frac{t}{Bc} \sinh (cx) \tag{56}$$

$$h_t = t + h_a e^{Bc} - \frac{t}{Bc} \sinh (Bc) \tag{57}$$

At the landside levee toe, $x = B$ and

$$\frac{dh}{dx} = \frac{H - h_t}{\overline{X} + L_2} = \frac{t}{B} + h_a c\ e^{Bc} - \frac{t}{B} \cosh (Bc) \tag{58}$$

and thus

$$H = h_t + (\overline{X} + L_2) \left[\frac{t}{B} + h_a c\ e^{Bc} - \frac{t}{B} \cosh (Bc) \right] \tag{59}$$

The uplift safety factor for the combined berm and top blanket at point x is given by Equation 38. At the landside levee toe $(x = B)$

$$F_B = \frac{Z_b \gamma_b' + t\gamma_t'}{(h_t - t)\gamma_w} \tag{60}$$

Setting $h_t - t = 0$, F_B approaches infinity and the maximum required value of t is

$$t(\text{maximum}) = \frac{h_a Bc\ e^{Bc}}{\sinh(Bc)} \tag{61}$$

If t is larger than that given by Equation 61, the piezometric profile will lie partly below the upper surface of the berm and the solution developed above will not apply.

Examples

13. Two cases are presented below--the first for $\overline{K} = \infty$ and the second for $\overline{K} = 0.5$. The same data are used as given in the main report and are repeated below:

$$H = 30 \text{ ft} , \quad Z_b = 5 \text{ ft} , \quad D = 50 \text{ ft}$$

$$k_f/k_b = 200 , \quad L_2 = 200 \text{ ft}$$

$$L_1 = 500 \text{ ft} , \quad \gamma_b' = \gamma_t' = \gamma_w , \quad F_B = F_o = 1.5 , \quad F_x \neq 1.5$$

$$c = \frac{1}{\sqrt{200 \times 5 \times 50}} = \frac{1}{223.6 \text{ ft}}$$

$$L_{LS} = \frac{1}{c} = 223.6 \text{ ft}$$

$$\underline{\overline{X}} = 223.6 \text{ ft} \tanh\left(\frac{500}{223.6}\right) = 218.5 \text{ ft}$$

$$h_a = \frac{Z_b \gamma_b'}{F_o \gamma_w} = \frac{5 \times 62.4}{1.5 \times 62.4} = \frac{10}{3} \text{ ft}$$

Condition A, $\overline{K}$ approaches infinity

14. Use Equations 57, 59, and 60. The uplift safety factor at the landside levee toe is

$$F_B = \frac{Z_b \gamma_b' + t\gamma_t'}{(h_t - t)\gamma_w} = \frac{5 + t}{h_t - t}$$

$$h_t = t + \frac{10}{3} e^{Bc} - \frac{t}{Bc} \sinh (Bc)$$

$$H = h_t + 418.5\left[\frac{10}{3} \times \frac{1}{223.6} e^{Bc} - \frac{t}{B} \cosh (Bc)\right]$$

Use a hand-held programmable calculator. The following tabulation is for B set at 320 ft:

t , ft	H , ft, cal.	F_B	h_t , ft
5.00	30.238	1.418	12.054
5.10	30.042	1.460	12.017
5.20	29.845	1.505	11.979
5.15	29.944	1.482	11.998
5.18	29.885	1.496	11.986
5.19	29.865	1.500	11.983

This tabulation is for B set at 325 ft:

t , ft	H , ft, cal.	F_B	h_t , ft
5.20	30.510	1.452	12.226
5.30	30.310	1.496	12.187
5.40	30.109	1.541	--
5.35	30.209	1.518	--
5.33	30.249	1.509	--
5.32	30.269	1.505	--
5.31	30.289	1.500	12.183

By linear interpolation,

$$\frac{B - 320}{325 - 320} = \frac{30.000 - 29.865}{30.289 - 29.865}$$

$$B = 321.6 \text{ ft} \text{ , say } 322 \text{ ft}$$

$$\frac{t - 5.19}{5.31 - 5.19} = \frac{321.6 - 320.0}{325.0 - 320.0}$$

$$t = 5.23 \text{ ft}$$

Check for $B = 321.6$ ft , $t = 5.23$ ft , $H = 30.0$ ft , $F_B = 1.501$, and $h_t = 12.046$ ft. The variation of uplift safety factor F_x and uplift h_x , using the data derived above and using Equations 38 and 56 are as tabulated on the following page:

x , ft	F_x(combined)	t_x , ft
0	1.50	0.00
50	1.74	0.81
100	1.88	1.63
150	1.91	2.44
200	1.85	3.25
250	1.73	4.07
300	1.58	4.88
321.6	1.50	5.23

15. The main report presents an analytical method for determining the dimensions and uplift safety factors for a berm having an infinite vertical permeability coefficient and a horizontal permeability coefficient equal to zero. The equations are given on pages 14 to 23 of the main report for $F_o = F_x = F_B = 1.50$. The berm width is 304 ft and t is 5.03 ft. The volume of the berm per station is 2347 cu yd. The volume of the berm having a constant outer slope with $B = 321.6$ ft and $t = 5.23$ ft , $b = 0$ is 3115 cu yd per station. Thus, the volume of the latter berm is 32.7 percent larger than that for the former berm.

Condition B, $\bar{K} = 0.5$

16. Using Equation 11

$$b = \frac{5(1 - 0.5)}{1 + 1.5} = 1.0 \text{ ft}$$

Using Equations 31 and 32

$$C_1 = \left(\frac{10}{3} - 1\right) K_o(\omega_o) + \left[\left(\frac{10}{3} - \frac{t - 1}{B}\, 223.6\right)\sqrt{1 + \frac{1}{5 \times 0.5}}\right] K_1(\omega_o)$$

$$C_2 = \left(\frac{10}{3} - 1\right) I_o(\omega_o) - \left[\left(\frac{10}{3} - \frac{(t - 1)223.6}{B}\right)\sqrt{1 + \frac{1}{5 \times 0.5}}\right] I_1(\omega_o)$$

The value of ω_o is obtained using Equations 17, 21 and 26.

$$\omega_o = \frac{2Z_b Bc\overline{K}}{t - b}\sqrt{1 + \frac{b}{Z_b\overline{K}}} = \frac{2 \times 5 \times B \times 0.5}{(t - 1)\sqrt{50,000}}\sqrt{1 + \frac{1}{5 \times 0.5}} = \frac{B\sqrt{3.5}}{(t - 1)\sqrt{5,000}}$$

ω_B is found using Equation 33:

$$\omega_B = \frac{2Z_b Bc\overline{K}}{t - b}\sqrt{1 + \frac{t}{Z_b\overline{K}}} = \frac{B\sqrt{2.5 + t}}{(t - 1)\sqrt{5000}}$$

Equations 35 and 37 are used to calculate h_t and H. With B set at 575 ft, the following results are tabulated:

Item	t = 8.0 ft	t = 8.5 ft	t = 9.0 ft
ω_o (at x = 0)	2.173296	2.028409	1.901634
$K_o(\omega_o)$	0.09220	0.1100	0.1285
$K_1(\omega_o)$	0.1116	0.1329	0.1593
$I_o(\omega_o)$	2.5787	2.3254	2.1301
$I_1(\omega_o)$	1.5677	1.6333	1.4505
ω_B (at x = B)	3.764259	3.595985	3.447004
$\zeta_1(\omega_B)$	7.881	6.768	5.916
$K_1(\omega_B)$	0.01637	0.01989	0.02366
$I_o(\omega_B)$	9.231	8.000	7.0571
$K_o(\omega_B)$	0.01454	0.01758	0.02081
h_t(ft)	17.064	16.670	16.343
H(ft)	31.313	30.140	29.132
F_B (at x = B)	1.158	1.346	1.566

Find F_B and t for $H = 30.0$ by interpolation as follows:

$$\frac{F_B - 1.346}{1.566 - 1.346} = \frac{30.140 - 30.000}{30.140 - 29.132}, \quad F_B = 1.38$$

$$\frac{t - 8.5}{9.5 - 8.5} = \frac{30.140 - 30.000}{30.140 - 29.132}$$

$$t = 8.57 \text{ ft}$$

At $B = 650$ ft for $H = 30.0$ ft the results were $F_B = 2.067$ and

$t = 11.24$ ft . At $B = 590$ ft for $H = 30.0$ ft the results were $F_B = 1.500$ and $t = 9.13$ ft . Using Equation 39, find the uplift safety factor for the berm only at selected values of x when $B = 590$ ft and $t = 9.13$ ft . To find ω_x use Equations 17 and 21 as follows:

$$\omega_x = 2\sqrt{\psi\zeta_x} = \frac{2Z_b Bc\bar{K}}{t - b}\sqrt{1 + \frac{(t - b)x}{Z_b B\bar{K}} + \frac{b}{Z_b\bar{K}}}$$

$$= \frac{10 \times 590 \times 0.5}{(9.13 - 1)\sqrt{50,000}}\sqrt{1 + \frac{(913 - 1)x}{5 \times 590 \times 0.5} + \frac{1}{5 \times 0.5}}$$

$$= 1.62273\sqrt{1.4 + 0.005512x}$$

Use Equation 22 to find h as follows:

$$h = \frac{t - b}{B} x + b + \omega_x\left[C_1 I_1(\omega_x) + C_2 K_1(\omega_x)\right]$$

$$= \frac{(9.13 - 1)}{590} x + 1 + \omega_x\left[C_1 I_1(\omega_x) + C_2 K_1(\omega_x)\right]$$

Use Equation 26 to find ω_o at $x = 0$:

$$\omega_o = \frac{2Z_b Bc\bar{K}}{t - b}\sqrt{1 + \frac{b}{Z_b\bar{K}}} = \frac{2 \times 5 \times 590 \times 0.5}{(9.13 - 1)\sqrt{50,000}}\sqrt{1 + \frac{1}{5 \times 0.5}} = 1.9200$$

Use Equations 31 and 32 to find C_1 and C_2 as shown below:

$$C_1 = \left(\frac{10}{3} - 1\right) K_o(1.92) + \left[\left(\frac{10}{3} - \frac{8.13\sqrt{50,000}}{590}\right)\sqrt{1 + \frac{1}{5 \times 0.5}}\right] K_1(1.92)$$

$$= \frac{7}{3} \times 0.1257 + 0.298408 \times 0.1555 = 0.3397$$

$$C_2 = \left(\frac{10}{3} - 1\right) I_o(1.92) - \left[\left(\frac{10}{3} - \frac{8.13\sqrt{50,000}}{590}\right)\sqrt{1 + \frac{1}{5 \times 0.5}}\right] I_1(1.92)$$

$$= \frac{7}{3} \times 2.157 - 0.298408 \times 1.476 = 4.5925$$

Use Equation 39 to find the uplift safety factor for the berm only at x as follows:

$$F_x = \frac{(Z_b\bar{K} + t_x)\gamma_t'}{(h - t_x)\gamma_w} = \frac{5 \times \frac{1}{2} + t_x}{h - t_x} = \frac{2.5 + t_x}{h - t_x}$$

Using the above equation, the following values may be derived:

x , ft	t_x , ft	ω_x	$I_1(\omega_x)$	$K_1(\omega_x)$	h	F_x
0	1.00	1.9200	1.4758	0.1555	3.33	1.50
100	2.38	2.2667	2.0349	0.09908	4.98	1.88
200	3.76	2.5670	2.6743	0.06800	6.89	2.00
300	5.13	2.8356	3.4088	0.04895	9.05	1.95
400	6.51	3.0809	4.2524	0.03645	11.48	1.81
500	7.89	3.3081	5.2189	0.02785	14.18	1.65
590	9.13	3.5000	6.2058	0.02224	16.87	1.50

Discussion

17. A mathematical solution for the seepage up through a landside berm is presented for a berm having a constant outer slope. The equations are explicit for the determination of the uplift head at the base of the top blanket, the calculated head H on the system, and the safety factor against uplift. It is desired to determine the berm width B and the berm thickness t at the landside levee toe. The equations are implicit for B and t and the solution must be obtained by trial.

18. The required dimensions of the berm are sensitive to the permeability rates as indicated by the curve of Figure 3 for conditions

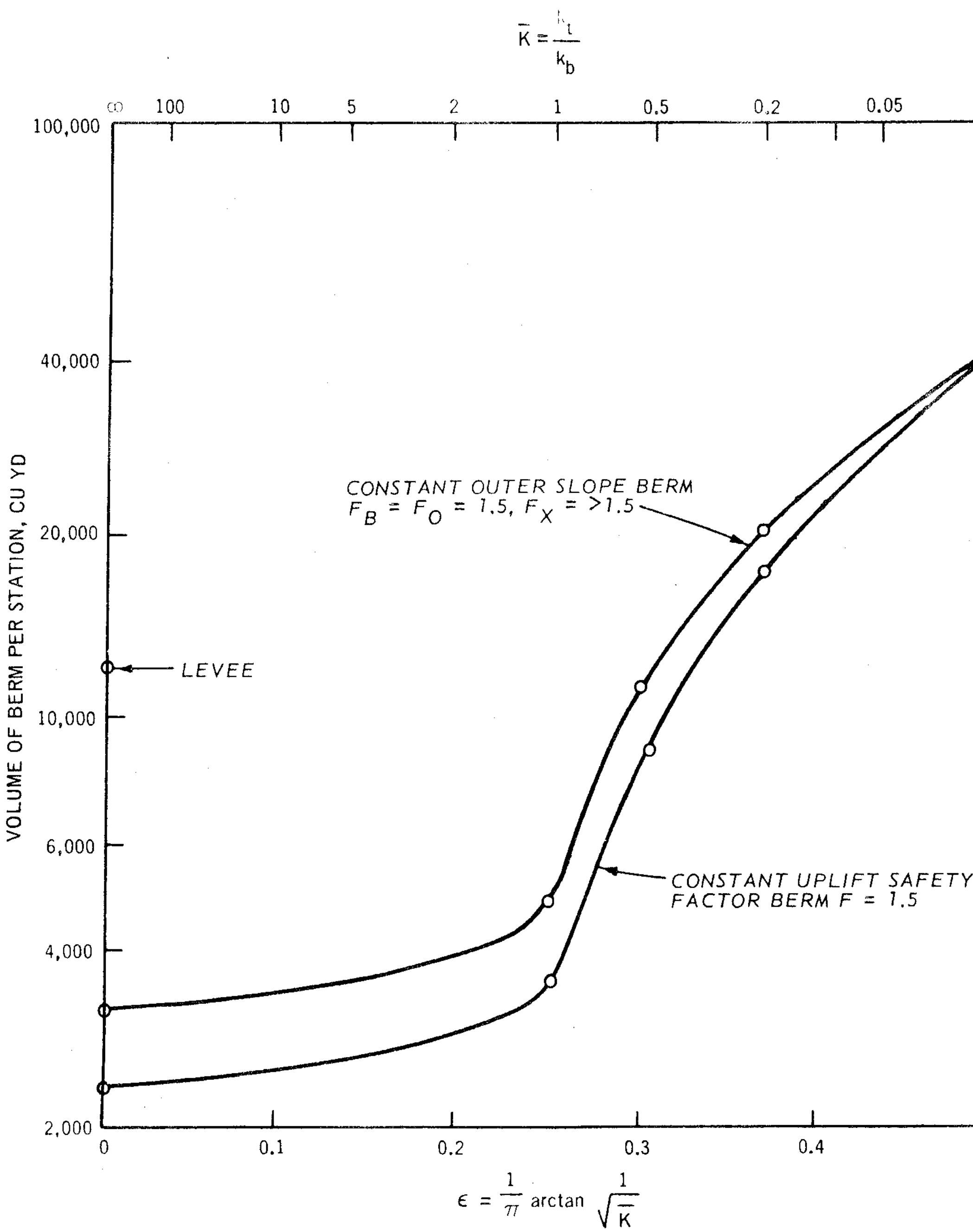

Figure 3. Volume of berm per station versus $\bar{K}$

where $\overline{K}$ is less than one. This sensitivity is rather minor when $\overline{K}$ is greater than one. As shown on Figure 3, the constant slope berm requires more fill for the same uplift safety factor at the berm toe and landside levee toe than a berm having a constant safety factor. The uplift safety factors at points between the landside levee toe and the berm toe for the constant slope berm are larger than those at the berm toe and levee toe. This is a result of the extra material required for this type of berm as compared to one with a constant safety factor. Thus, the constant slope berm is more costly than the berm with a constant safety factor. The variation of the volume of fill per station of a berm having a constant outer slope as compared with a berm having a constant uplift safety factor is shown in Figure 4. The variation of the berm width and thickness at the landside levee toe is also shown in Figure 4. Two examples are given--one where $\overline{K}$ is infinite and the other with $\overline{K}$ equal to 0.5.

Conclusions

19. The following conclusions are made:

   a. This analysis demonstrates that a berm having a constant outer slope requires more fill and therefore is more expensive than a berm having a constant uplift safety factor equal in value to the design value of the constant slope berm.

   b. The analysis also indicates, as shown in Figure 3, that the volume of a seepage berm is very sensitive to the permeability ratio $\overline{K}$ when the ratio is less than one. Thus the most pervious fill available should be used for the berm.

   c. The conclusions of the main report remain unchanged.

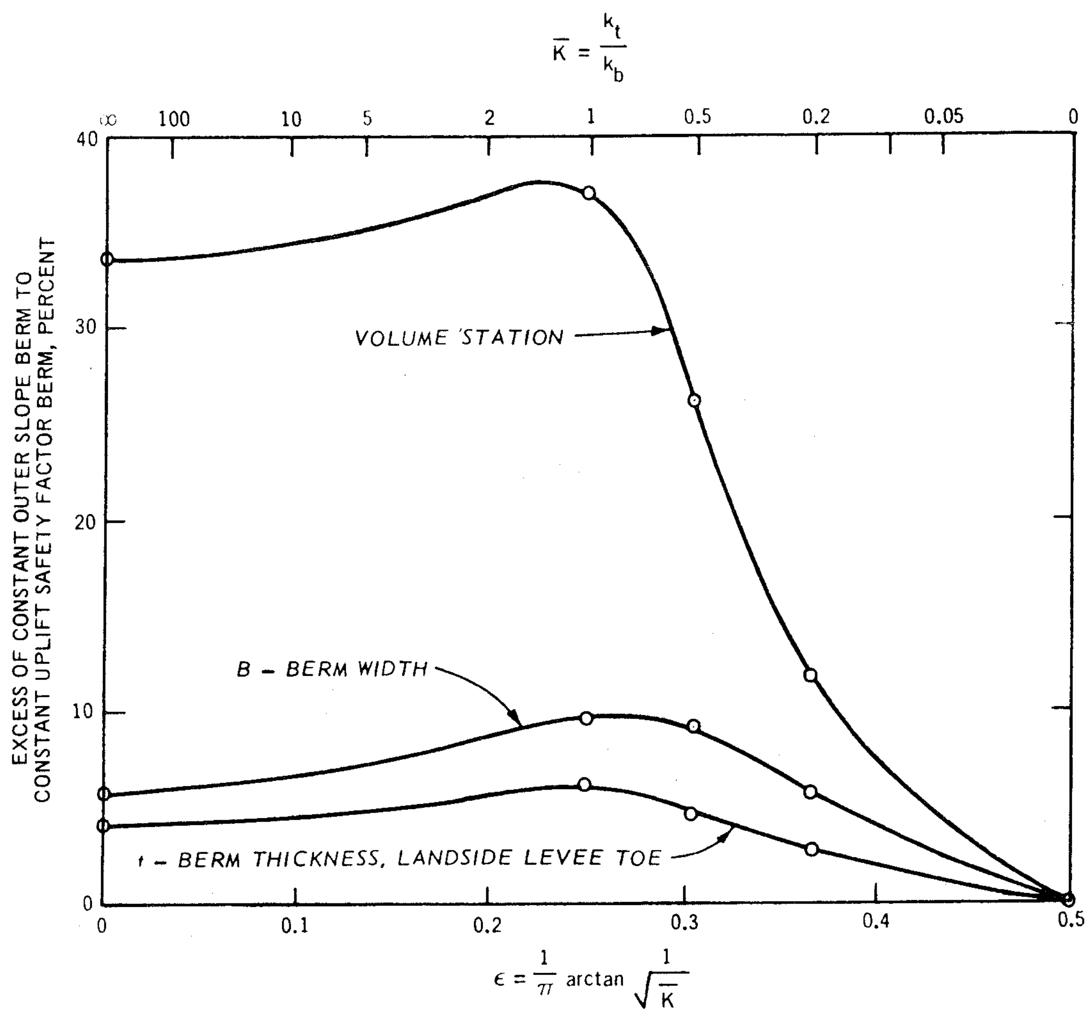

Figure 4. Percent excess of volume/station, berm witdth, and berm thickness at landside levee toe

<u>Appendix A: Notation</u>

b	Thickness of berm at the landside toe
B	Seepage berm width from the landside levee toe to the landside berm toe
c	$(k_b/k_f D Z_b)^{1/2}$
C_1, C_2, C_3, C_4, C_5, C_6	Constants
D	Pervious foundation thickness
e	2.71828
F_B	Uplift safety factor at landside levee toe
F_o	Uplift safety factor at landside berm toe
F_x	Uplift safety factor at point x
h	Seepage uplift head at base of top blanket at point x, referenced to top of top blanket
h_a	Allowable seepage uplift head at the landside berm toe (measured at base of top blanket referenced to top of top blanket)
H	Net hydraulic head between river flood level and the landside upper surface of the top blanket
i	Seepage gradient
$I_o(\ )$	Modified Bessel function, first kind, zero order
$I_1(\ )$	Modified Bessel function, first kind, first order
k_b	Vertical permeability coefficient of top blanket
k_f	Horizontal coefficient of permeability of the previous foundation
k_t	Vertical permeability coefficient of berm
$\overline{K}$	Permeability ratio of berm to top blanket, equals k_t/k_b

$K_o(\ )$ Modified Bessel function, second kind, zero order

$K_1(\ )$ Modified Bessel function, second kind, first order

L_1 Width of riverside top blanket measured from riverside levee toe to riverbank

L_2 Base width of levee

L_{LS} Effective length of landside top blanket measured landward from berm toe

n An integer which takes all values from one to infinity (see Appendix B of this supplement)

$n!$ Factorial number equal to $1 \times 2 \times 3 \times 4 \times 5 \ldots \times n$

t Maximum berm thickness at landside levee toe

t_x Berm thickness at point x

x Horizontal distance riverward of landside berm toe; may also be a mathematical variable (see Appendix B)

$\underline{\overline{X}}$ Effective length of riverside top blanket

y Piezometric head above top of seepage berm

Z_b Thickness of top blanket

γ'_b Buoyant unit weight of top blanket

γ'_t Buoyant unit weight of berm

γ_w Unit weight of water

ε Equal to $1/\pi \left(\arctan 1/\sqrt{\overline{K}}\right)$

ζ A variable (see Equation 17)

Σ Summation sign

ψ A constant (see Equation 21)

ω_B A constant (see Equation 27)

ω_o	A constant (see Equation 26)
ω_x	A variable (see Equation 25)
π	A constant, rounded to 3.1416

Appendix B: Bessel Functions

The modified Bessel function can be expressed in terms of the infinite series given below. These can be programmed on hand calculators:

$$I_o(x) = \left[1 + \sum_{n=1}^{n=\infty} \left(\frac{(Z/2)^n}{n!}\right)^2\right]$$

$$I_1(x) = \frac{x}{2}\left[1 + \sum_{n=1}^{n=\infty} \frac{(Z/2)^{2n}}{n!(n+1)!}\right]$$

$$K_o(x) = \left\{\left[I_o(x)\right] \times \left[-\ln x + 0.115932\right] + \sum_{n=1}^{n=\infty} \frac{\left(1 + \frac{1}{2} + \frac{1}{3} \dots \frac{1}{n}\right)\left(\frac{x}{2}\right)^{2n}}{(n!)^2}\right\}$$

$$K_1(x) = \frac{1}{x} + \left[I_1(x)\right] \ln \frac{x}{2} - \frac{x}{4}\left[\psi(1) + \psi(2)\right]$$

$$- \frac{x}{4} \sum_{1}^{\infty} \left[\psi(n+1) + \psi(n+2)\right] \frac{\left(\frac{x}{2}\right)^{2n}}{n!(n+1)!}$$

where

$$\psi(1) + \psi(2) = -2\gamma + 1 = -0.154431$$

$$\psi(n+1) + \psi(n+2) = -2\gamma + \frac{1}{n+1} + 2\sum_{1}^{n} \frac{1}{n}$$

$$\gamma = 0.577216$$

Supplement No. 2

RIVERSIDE SEEPAGE BERMS

Contents

MATHEMATICAL ANALYSES OF LANDSIDE SEEPAGE BERMS

RIVERSIDE SEEPAGE BERMS

Introduction

1. This report is Supplement No. 2 to Technical Report REMR-GT-1 (referred to as the main report). This report is concerned with riverside seepage berms. While the main report is concerned with landside berms, this report supplements the main report so that information on mathematical analyses of seepage berms may be combined under one cover, and is a condensation of a report completed under Purchase Order DACW39-75-M-4085, US Army Engineer Waterways Experiment Station, dated June 1975.

Assumptions

2. The assumptions are those of the main report.

Case X, natural riverside blanket

3. A sketch of the case is shown in Figure 1. The far riverside top blanket is assumed to be impervious. This is a result of a transformation of conditions riverside of $x = L_1$. If there are complex conditions, each zone is transformed and an effective length R_o found. For each step, a new R_o is determined and that shown in Figure 1 is the last step prior to finding X_1.

4. The head loss through the riverside top blanket S is related to seepage through the pervious foundation:

$$k_f D \frac{d^2S}{dx^2} = \frac{k_b}{Z_b} S \tag{1}$$

Dividing both sides of Equation 1 by $k_f D$ gives

$$\frac{d^2S}{dx^2} = \frac{k_b}{k_f D Z_b} S = c^2 S \tag{2}$$

where

$$c^2 = \frac{k_b}{k_f D Z_b}$$

The solution of Equation 2 is

$$S = C_1 e^{cx} + C_2 e^{-cx} \tag{3}$$

At $x = L_1$ the boundary conditions are

$$S = S_o = C_1 e^{cL_1} + C_2 e^{-cL_1} \tag{4}$$

Also

$$\frac{dS}{dx} = -\frac{S_o}{R_o} = c\left(C_1 e^{cL_1} - C_2 e^{-cL_1}\right) \tag{5}$$

at $x = 0$, $S = S_t$; thus,

$$S_t = C_1 + C_2 \tag{6}$$

$$C_1 = \frac{S_o}{2}\left(1 - \frac{1}{cR_o}\right)e^{-cL_1} \tag{7}$$

$$C_2 = \frac{S_o}{2}\left(1 + \frac{1}{cR_o}\right)e^{cL_1} \tag{8}$$

and by algebraic manipulation,

$$S = S_o\left\{\cosh\left[c(L - x)\right] + \frac{1}{cR_o} \sinh \left[c(L - x)\right]\right\} \tag{9}$$

and

$$\frac{dS}{dx} = -cS_o \left\{ \sinh\left[c(L - x)\right] + \frac{1}{cR_o} \cosh\left[c(L - x)\right] \right\} \tag{10}$$

at $x = 0$, $S = S_t$, $dS/dx = -S_t/X_1$. The effective length of the entire riverside system is

$$X_1 = \frac{-S_t}{\frac{dS}{dx}} = \frac{-S_o\left[\cosh\left(cL_1\right) + \frac{1}{cR_o}\sinh\left(cL_1\right)\right]}{-cS_o\left[\sinh\left(cL_1\right) + \frac{1}{cR_o}\cosh\left(cL_1\right)\right]} = \frac{1}{c}\left[\frac{cR_o + \tanh\left(cL_1\right)}{1 + cR_o \tanh\left(cL_1\right)}\right] \tag{11a}$$

If $R_o = 0$

$$X_1 = \frac{\tanh(cL_1)}{c} \tag{11b}$$

If $R_o \to \infty$

$$X_1 = \frac{1}{c \tanh(cL_1)} \tag{11c}$$

If $R_o = 1/c$

$$X_1 = \frac{1}{c} \tag{11d}$$

For example:

$$\frac{k_f}{k_b} = 200\,, \quad Z_b = 5 \text{ ft}, \quad D = 50 \text{ ft}, \quad L_1 = 500 \text{ ft},$$

$$R_o = 223.6 \text{ ft}, \quad c = \frac{1}{223.6} \text{ ft}$$

$$X_1 = 223.6\left(\frac{\frac{223.6}{223.6} + \tanh\frac{500}{223.6}}{1 + \frac{223.6}{223.6}\tanh\frac{500}{223.6}}\right) = 223.6 \text{ ft}$$

Case XI, constructed
berm of uniform thickness

5. This berm is constructed over a natural blanket, as shown in Figure 2. The conditions riverside of L_1 have been reduced to an

effective length of impervious blanket of length R_o. The thickness of the natural top blanket Z_b is converted to Z_b' so that the constructed berm permeability k_t may be used for both the berm and the transformed top blanket thickness:

$$Z_b' = Z_b \frac{k_t}{k_b} = Z_b \overline{K} \tag{12}$$

Equation 2 for this case is

$$\frac{d^2 S}{dx^2} = \frac{k_t S}{k_f D (Z_B + Z_b \overline{K})} = \frac{k_t S}{k_f D \left(\frac{Z_B}{Z_b} + \overline{K} \right)} \frac{k_b}{k_b Z_b} = \frac{c^2 \overline{K} S}{\left(\frac{Z_B}{Z_b} + \overline{K} \right)} = \theta^2 S \tag{13}$$

where

$$\theta^2 = \frac{c^2}{1 + \frac{Z_B}{Z_b \overline{K}}} \tag{14}$$

The solution for Equation 13 using the results of the previous case, is

$$S = S_o \left\{ \cosh \left[\theta (L_1 - x) \right] + \frac{1}{\theta R_o} \sinh \left[\theta (L_1 - x) \right] \right\} \tag{15}$$

$$X_1 = \frac{1}{\theta} \left[\frac{\theta R_o + \tanh \theta L_1}{1 + \theta R_o \tanh \theta L_1} \right] \tag{16a}$$

If $k_t = 0$, $\overline{K} = 0$, $\theta = 0$

$$\frac{\tanh (\theta L_1)}{\theta} \rightarrow \frac{\theta L_1}{\theta} \rightarrow L_1 \tag{16b}$$

$$X_1 \rightarrow R_o + L_1 \tag{16c}$$

If $Z_B = 0$, $\theta = c$ and Equation 16a becomes Equation 11a. For example:

$\frac{k_f}{k_b} = 200$, $\frac{k_t}{k_b} = \overline{K} = \frac{1}{10}$, $Z_b = 5$ ft, $D = 50$ ft,
$L_1 = 500$ ft, $R_o = 223.6$ ft, $\frac{1}{c} = 223.6$ ft, $Z_B = 5.5$ ft,

$$\theta = \frac{c}{\sqrt{1 + \frac{Z_B}{Z_b \overline{K}}}} = \frac{1}{223.6 \sqrt{1 + \frac{5.5 \times 10}{5}}} = \frac{1}{774.6 \text{ ft}}$$

$$X_1 = 774.6 \left(\frac{\frac{223.6}{774.6} + \tanh \frac{500}{774.6}}{1 + \frac{223.6}{774.6} \tanh \frac{500}{774.6}} \right) = 570.4 \text{ ft}$$

Case XII, constructed trapezoidal berm

6. This berm is constructed over a natural top blanket, as shown in Figure 3. The natural top blanket riverside of $x = L_1$ has been reduced to an effective length R_o of an impervious top blanket. The thickness t_x of the trapezoidal blanket at point x is

$$t_x = t - [t - b] \frac{x}{L_1} \tag{17}$$

The basic second-order differential equation for this case is

$$\frac{d^2 S}{dx^2} = \frac{k_t}{k_f D} \frac{S}{(t_x + Z_b \overline{K})} = \frac{k_t}{k_f D} \frac{S}{\left[t - (t - b) \frac{x}{L_1} + Z_b \overline{K} \right]} \frac{k_b}{k_b}$$

$$= \frac{k_b S}{k_f D Z_b \left[\frac{t}{Z_b} - \frac{(t - b)x}{Z_b L_1} + \overline{K} \right]} \frac{k_t}{k_b} = \frac{c^2 S \overline{K}}{y} \tag{18}$$

where

$$\frac{t}{Z_b} - \frac{(t - b)x}{Z_b L_1} + \overline{K} = y \tag{19}$$

t = the berm thickness at $x = 0$ (riverside levee toe)

b = the berm thickness at $x = L_1$ (riverside berm toe)

Differentiate Equation 19 to obtain

$$dx = \frac{-Z_b L_1}{t - b} dy$$

so that Equation 18 becomes

$$\frac{d^2S}{dy^2} = \left(\frac{Z_b L_1 c}{t - b}\right)^2_{\overline{K}} \frac{S}{y} = \frac{\zeta S}{y} \tag{20a}$$

where

$$\zeta = \left(\frac{Z_b L_1 c}{t - b}\right) \tag{20b}$$

The solution of Equation 20a is

$$S = \omega_x \left[C_1 I_1(\omega_x) + C_2 K_1(\omega_x)\right] \tag{21}$$

where $I_1(\ )$ and $K_1(\ )$ are modified Bessel functions of the first and second kind of order one and

$$\omega_x = \frac{2Z_b L_1 c\overline{K}}{t - b} \sqrt{\frac{t}{Z_b\overline{K}} - \frac{(t - b)x}{Z_b L_1 \overline{K}} + 1} \tag{22}$$

$$\frac{dS}{dy} = \frac{\omega_x^2}{2y} \left[C_1 I_o(\omega_x) - C_2 K_o(\omega_x)\right] \tag{23}$$

where $I_o(\ )$ and $K_o(\ )$ are modified Bessel functions of the first and second kind of zero order. The values of C_1 and C_2 are determined by the boundary conditions at $x = L_1$, where $S = S_o$ and $dS/dx = -S_o/R_o$:

$$\frac{C_1}{S_o} = K_o(\omega_o) + \frac{\sqrt{1 + \frac{b}{Z_b\overline{K}}}}{R_o c} K_1(\omega_o) \tag{24}$$

$$\frac{C_2}{S_o} = I_o(\omega_o) - \frac{\sqrt{1 + \frac{b}{Z_b\overline{K}}}}{R_o c} I_1(\omega_o) \qquad (25)$$

$$\omega_o = \frac{2Z_b L_1 c\overline{K}}{t - b} \sqrt{1 + \frac{b}{Z_b\overline{K}}} \qquad (26)$$

The effective length X_1 of the entire riverside top blanket, constructed blanket, and pervious foundation is

$$X_1 = \frac{1}{c} \sqrt{1 + \frac{t}{Z_b\overline{K}}} \, \frac{\left[I_1(\omega_t) + \frac{C_2}{C_1} K_1(\omega_t)\right]}{\left[I_o(\omega_t) - \frac{C_2}{C_1} K_o(\omega_t)\right]} \qquad (27)$$

where

$$\omega_t = \frac{2Z_b L_1 c\overline{K}}{t - b} \sqrt{1 + \frac{t}{Z_b\overline{K}}} \qquad (28)$$

If the natural top blanket is missing and $b = 0$, then

$$X_1 = \sqrt{\frac{k_f Dt}{k_t}} \, \frac{I_1\left(2L_1 \sqrt{\frac{k_t}{k_f Dt}}\right)}{I_o\left(2L_1 \sqrt{\frac{k_t}{k_f Dt}}\right)} \qquad (29)$$

Example:

$\frac{k_t}{k_b} = \frac{1}{10}$, $\frac{k_b}{k_f} = \frac{1}{200}$, $Z_b = 5$ ft, $D = 50$ ft

$b = 1$ ft, $t = 10$ ft, $L_1 = 500$ ft, $R_o = 223.6$ ft $= \frac{1}{c}$

Find X_1 the effective length of the riverside system:

$$\omega_o = \frac{2Z_b L_1 c\bar{K}}{t - b} \sqrt{1 + \frac{b}{Z_b\bar{K}}} = \frac{2 \times 5 \times 500}{(10 - 1)223.6 \times 10} \sqrt{1 + \frac{10}{5}} = 0.430344$$

$$\omega_t = \frac{2Z_b L_1 c\bar{K}}{t - b} \sqrt{1 + \frac{t}{Z_b\bar{K}}} = \frac{2 \times 5 \times 500}{(10 - 1)223.6 \times 10} \sqrt{1 + \frac{100}{5}} = 1.138585$$

The required modified Bessel functions are:

$I_o(\omega_o) = 1.0469$	$I_o(\omega_t) = 1.3514$
$I_1(\omega_o) = 0.2202$	$I_1(\omega_t) = 0.6666$
$K_o(\omega_o) = 1.0511$	$K_o(\omega_t) = 0.3466$
$K_1(\omega_o) = 1.9528$	$K_1(\omega_t) = 0.4790$

$$\frac{C_2}{C_1} = \frac{I_o(\omega_o) - \sqrt{3}\, I_1(\omega_o)}{K_o(\omega_o) + \sqrt{3}\, K_1(\omega_o)} = 0.150109$$

$R_o = 1/c$, therefore $R_o c = 1$

$$X_1 = \frac{1}{c} \sqrt{1 + \frac{t}{Z_b\bar{K}}} \left[\frac{I_1(\omega_t) + \frac{C_2}{C_1} K_1(\omega_t)}{I_o(\omega_t) - \frac{C_2}{C_1} K_o(\omega_t)} \right]$$

$$= 223.6 \sqrt{1 + \frac{10 \times 10}{5}} \left[\frac{0.6666 + 0.1501 \times 0.4790}{1.3514 - 0.1501 \times 0.3466} \right] = 582.4 \text{ ft}$$

Case XIII, constructed
berm having a constant gradient

7. A sketch for this case is shown in Figure 4. The basic second order differential equation is

$$\frac{d^2S}{dx^2} = \frac{k_t S}{k_f D(t_x + Z_b \bar{K})} = \frac{ik_t}{k_f D} \tag{30}$$

where i , the constant seepage gradient in the combined top blanket and berm, is

$$i = \frac{S}{t_x + Z_b \bar{K}} \tag{31}$$

The solution of Equation 30 is

$$S = \frac{ik_t}{k_f D} \frac{x^2}{2} + C_1 x + C_2 \tag{32}$$

At $x = L_1$ the boundary conditions are

$$S = S_o \ , \quad \frac{dS}{dx} = - \frac{S_o}{R_o}$$

The values of C_1 and C_2 are based upon these conditions:

$$C_1 = - \left(\frac{S_o}{R_o} + \frac{ik_t L_1}{k_f D} \right) \tag{33}$$

$$C_2 = S_o + \frac{S_o L_1}{R_o} + \frac{ik_t L_1^2}{2k_f D} \tag{34}$$

$$S = \frac{ik_t}{2k_f D} (L_1 - X)^2 + \frac{S_o}{R_o} (L_1 - x + R_o) \tag{35}$$

At $x = 0$, the boundary conditions are:

$$S = S_t \ , \quad \frac{dS}{dx} = - \frac{S_t}{X_1}$$

so that

$$X_1 = \frac{-S_t}{\frac{dS}{dx}} = \frac{-\left[\frac{ik_t L^2}{2k_f D} + \frac{S_o}{R_o}(L_1 + R_o)\right]}{-\left[\frac{ik_t L_1}{2k_f D} + \frac{S_o}{R_o}(L_1 + R_o)\right]} \tag{36}$$

At $x = L_1$ the gradient i through the berm and blanket is

$$i = \frac{S_o}{b + Z_b\overline{K}} \tag{37}$$

Inserting Equation 37 into Equation 36 results in

$$X_1 = \frac{\frac{L_1}{2} + \frac{b + Z_b\overline{K}}{c^2 Z_b L_1}\left(\frac{L_1}{R_o} + 1\right)}{1 + \frac{b + Z_b\overline{K}}{c^2 Z_b L_1 \overline{K} R_o}} \tag{38}$$

If $R_o = 0$ then it can be shown that the effective length is

$$X_1 = \frac{L_1}{2} \tag{39}$$

As an example:

$$\frac{k_b}{k_f} = \frac{1}{200}, \quad \frac{k_t}{k_b} = \overline{K} = \frac{1}{10}, \quad Z_b = 5 \text{ ft}, \quad D = 50 \text{ ft}, \quad L_1 = 500 \text{ ft}$$

$$R_o = \frac{1}{c} = 223.6 \text{ ft}$$

Set the cross-sectional area of the berm to $500[(1 + 10)/2] = 2750 \text{ ft}^2$.

$$t_x = \frac{c^2\overline{K}Z_b}{2}(L_1 - x)^2 + (b + Z_b\overline{K})\frac{(L_1 - x)}{R_o} + b$$

$$t = \frac{c^2\bar{K}Z_b}{2} L_1^2 + (b + Z_b\bar{K}) \frac{L_1}{R_o} + b$$

$$b = \frac{2AR_o}{L_1(L_1 + 2R_o)} - \frac{Z_b\bar{K}L_1}{L_1 + 2R_o} - \frac{C^2\bar{K}Z_bL_1^2R_o}{3(L_1 + 2R_o)}$$

$$t = \frac{1}{50,000} \times \frac{1}{10} \times \frac{5}{2} \times 500^2 + \left(b + \frac{5}{10}\right) \frac{500}{223.6} + b = 3.2361b + 2.36807$$

$b = 2.29$ ft, $t = 9.79$ ft

$$X_1 = \frac{\left[\frac{500}{2} + \frac{(2.29 + 5 \times 0.1)\ 50,000}{5 \times 500 \times \frac{1}{10}} \left(\frac{500}{223.6} + 1\right)\right]}{\left[1 + \frac{(2.29 \times 5 \times 0.1)\ 50,000}{5 \times 500 \times \frac{1}{10} \times 223.6}\right]}$$

$X_1 = 588.1$ ft

Case XIV, step approximation

8. The sketch for this case is shown in Figure 5, where the constructed berm has a complex cross-section. The cross-section is approximated by steps and risers which may or may not be equal in length or in riser increments. A solution is obtained for the effective length X_1 for the first step on the far riverside using R_o and Equation 16a. The procedure is repeated for the next step using X_1 found for R_o and a new value of X_1 is obtained. This procedure is repeated until X_1 for the entire system is found.

9. As an example, use the same data given for the trapezoidal berm example. The term "M" indicates the number of steps used and L_1/M indicates the length of each individual step. When M is 1, then the approximated step is that of a berm of uniform thickness:

M	L_1/M, ft	X_1, ft
1	500	570.426
5	100	581.931
10	50	581.947
25	20	581.548
50	10	581.324
100	5	581.192

The decrease in the effective length X_1 after $M = 10$ may be the result of truncation of figures in the hand calculator used in the computation. The results for $M = 10$ are very close to that developed in the example for the trapezoidal berm where $X_1 = 582.4$ ft.

Discussion

10. Mathematical solutions have been presented for various types of upstream berms and natural blankets. The solution given for a natural blanket permits the calculation for the effective length of the upstream system when the thickness and permeability of the top blanket are not uniform. The solutions of the trapezoidal berm and the berm with a constant seepage gradient are presented for cases where the berms have a finite thickness b at the upstream end. An approximate method is presented which is very powerful in that it permits the solution for berms having a complex cross section. It may be used for the trapezoidal and constant gradient berms if desired. In the examples given the berm width L_1 and cross-sectional areas are equal. The effective lengths X_1 are:

Berm, uniform thickness:	570.4 ft
Berm, trapezoid:	582.4 ft
Berm, constant gradient:	588.1 ft

Thus, a berm designed for a constant gradient is the most effective, but not excessively so.

11. No attempt has been made to find the optimum berm length for a given cross section. Limited studies made by the author indicated

that optimal lengths occur when the upstream berm toe thickness b is zero.

12. It was also found that the effective length X_1 is not very sensitive to variation in berm length. For practical cases the determination of the optimum berm length should include costs of clearing and foundation preparation of the berm area. Further it may not be desirable to build a berm with the thickness too small just to obtain an optimal berm width because of possible damage to the berm from erosion by runoff and riverflows.

Conclusion

13. In conclusion, it should be noted that the riverside berm should be constructed with the least pervious soils that are economically available, while landside berms should be constructed with the most pervious soils.

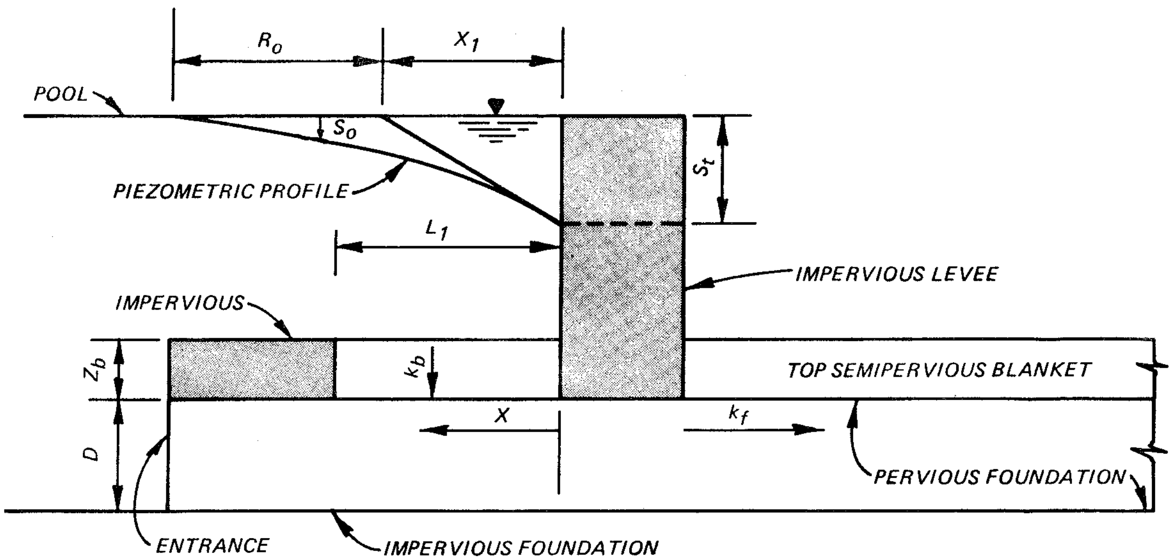

Figure 1. Natural riverside blanket

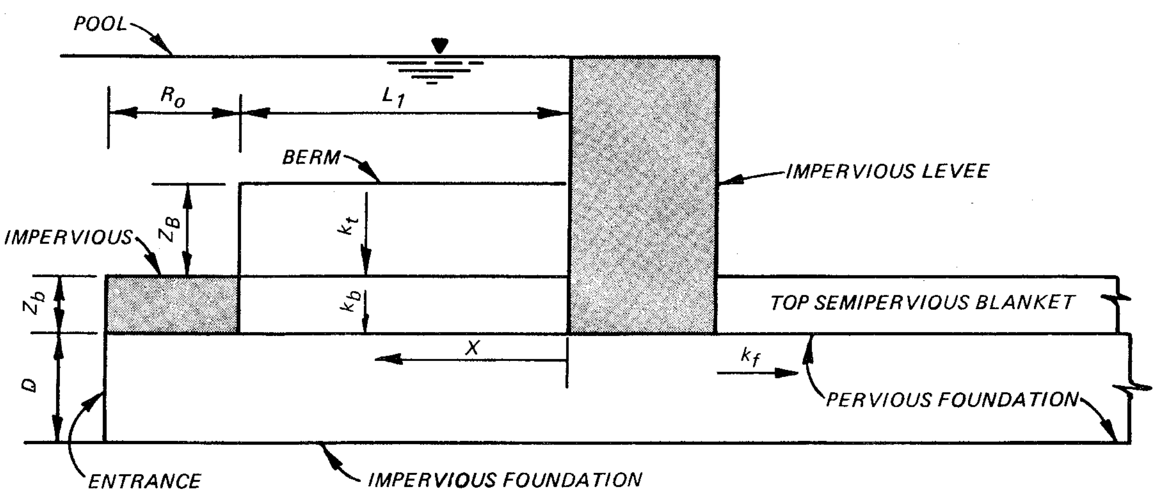

Figure 2. Constructed berm of uniform thickness

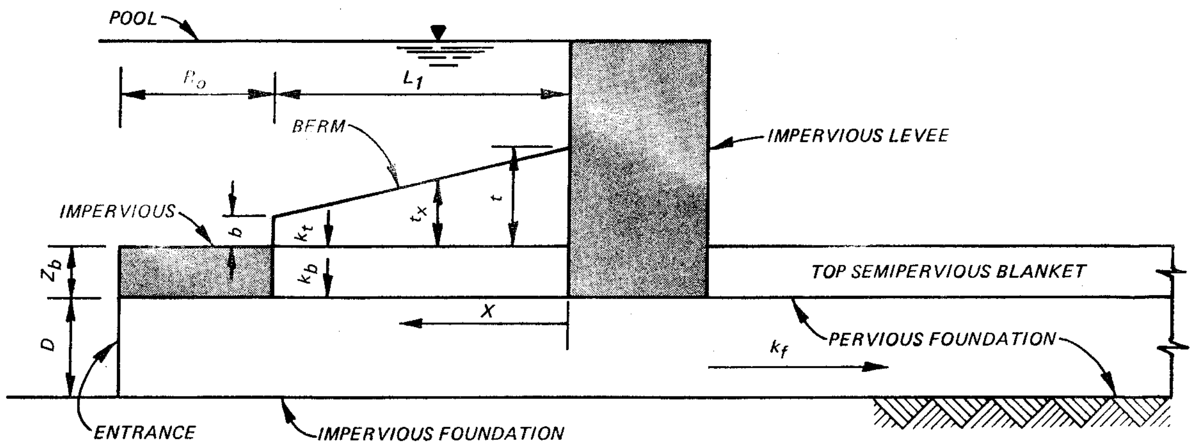

Figure 3. Constructed trapezoidal berm

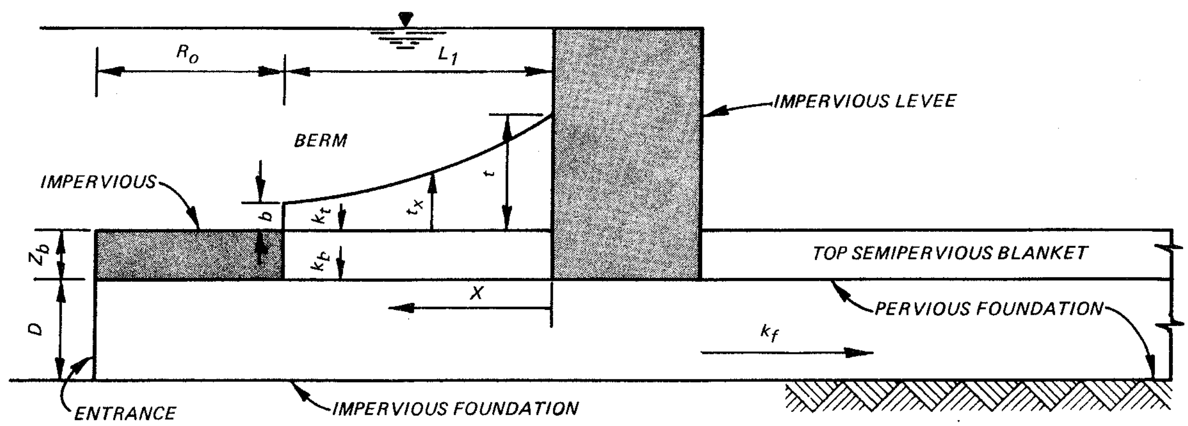

Figure 4. Constructed berm having a constant gradient

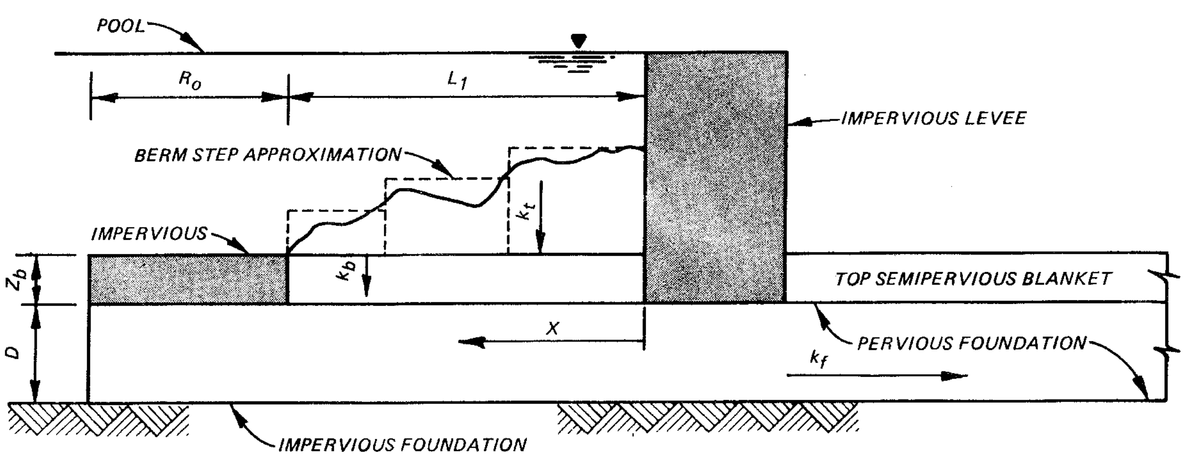

Figure 5. Step approximation

Appendix A: Notation

c	$(k_b/k_f D Z_b)^{1/2}$
$C_1, C_2, C_3, C_4, C_5, C_6$	Constants
D	Pervious foundation thickness
i	Seepage gradient (see Equation 31)
$I_o(\ )$	Modified Bessel function, first kind, zero order
$I_1(\ )$	Modified Bessel function, first kind, first order
k_b	Vertical permeability coefficient of top blanket
k_t	Vertical permeability coefficient of berm
$\overline{K}$	Permeability ratio of berm to top blanket, equals k_t/k_b
$K_o(\ )$	Modified Bessel function, second kind, zero order
$K_1(\ )$	Modified Bessel function, second kind, first order
L_1	Width of riverside top blanket measured from riverside levee toe to riverbank; width of riverside berm
M	Number of steps used
R_o	Effective length of riverside top blanket
S	Head loss through berm and top blanket
S_o	Head loss through berm and top blanket at $x = L_1$, riverside berm toe
S_t	Head loss through berm and top blanket at $x = 0$, riverside levee toe
t	Maximum berm thickness at landside levee toe
t_x	Berm thickness at point x
x	Horizontal distance riverward of landside berm toe; may also be a mathematical variable
X_1	Effective length of riverside berm and blanket system

y Piezometric head above top of seepage berm; a variable (see Equation 19)

Z_b Thickness of top blanket

Z'_b Transformed thickness of top blanket (see Equation 12)

Z_B Thickness of the constructed berm of uniform thickness

ζ A constant (see Equation 20b)

θ A constant (see Equation 14)

ψ A constant

ω_o A constant (see Equation 26)

ω_t A constant (see Equation 28)

ω_x A variable (see Equation 22)

Supplement No. 3

SOLUTIONS FOR GENERAL CASES AND SHORT BERMS

Contents

MATHEMATICAL ANALYSIS OF LANDSIDE SEEPAGE BERMS

SOLUTIONS FOR GENERAL CASES AND SHORT BERMS

Introduction

1. This supplement is a continuation of the work presented in Technical Report REMR-GT-1 (referred to as the main report) and extended in Supplement No. 1. In addition, analyses for short berms are also presented. The allowable uplift load h_a at the base of the top blanket just under the berm toe is

$$h_a = \frac{Z_b \gamma_b'}{F \gamma_w} \tag{1}$$

where

Z_b = the top semipervious blanket thickness

γ_b' = the buoyant unit weight of the top blanket

F = the uplift safety factor

γ_w = the unit weight of water

To generalize the solution, the uplift safety factor will be F_{LS} which is applied only to the top blanket at the berm toe and which may be different from the safety factor at the berm toe for the berm only or that for the combined berm and top blanket. Equation 1 becomes

$$h_a = \frac{Z_b \gamma_b'}{F_{LS} \gamma_w} \tag{2}$$

The origin of the coordinate systems for solutions presented in this report will be at the landside levee toe and will be positive in the levee landward direction.

Assumptions

2. The assumptions of the main report and Supplements No. 1 and No. 2 apply to this report. In some instances it may be desirable to

design berm widths (landward dimension) that are less than those given in the main report and Supplement No. 1. For these cases the computed uplift safety factors for the top blanket at the berm toe ($x = B$) will be less than unity. This indicates the need for additional assumptions which are:

a. The minimum uplift safety factor F_{LS} at the berm toe for the top blanket is unity.

b. A crack develops in the top blanket at and parallel to the berm toe.

3. These assumptions are justified as follows. As the uplift head h_a increases, a condition will develop where the transmissibility of the foundation is insufficient to conduct away the foundation seepage. The excess seepage will collect under the top blanket to form a seepage blister. The uplift safety factor for the top blanket will remain at unity. If the distortion of the top blanket becomes excessive, it will fail by cracking at the berm toe in a direction parallel to the berm toe. Seepage will emerge through the crack and the uplift safety factor of the top blanket will remain at unity. The seepage up through the crack may tend to form "boils" and possibly "pipes," but the duration of the flood may be such as not to permit the development of dangerous conditions. In some cases it may be desirable to install a trench drain along the berm toe. In some instances the top blanket will not be uniform and local boils may develop. The uplift safety factor for the top blanket will not be uniform but will be greater than unity. The value of F_{LS} at the berm toe for the top blanket is assumed to be equal to 1.0 for all short berms.

4. The foundation seepage flux per unit levee length at the berm toe is for $x = B - dx$:

$$\frac{Q_B}{k_f D} = - \frac{dh}{dx} \tag{3}$$

where Q_B equals seepage per unit length of levee in the pervious foundation at berm toe ($x = B$). At $x = B + dx$ the flux per unit length is

$$\frac{Q_{LS}}{k_f D} = h_a c = \frac{h_a}{L'_{LS}} \tag{4}$$

where

k_f = the horizontal permeability of the pervious foundation

D = the thickness of the pervious foundation

$c = \sqrt{\frac{k_b}{k_f D Z_b}} = \frac{1}{L_{LS}}$

k_b = the vertical permeability of the top blanket

Z_b = the thickness of the top blanket

L_{LS} = the effective length of the landside top blanket

If the seepage flux up through the crack in the top blanket at the berm toe is zero, then

$$\frac{Q_B}{k_f D} = \frac{Q_{LS}}{k_f D} = - \frac{dh}{dx} = h_a c \tag{5}$$

If the seepage flux up through the crack is not zero, then its value per unit length of berm is

$$\frac{Q_{BT}}{k_f D} = - \frac{dh}{dx} - h_a c \tag{6}$$

<u>Case I, Impervious Berms</u>

5. The basic second-order differential equation is

$$\frac{d^2 h}{dx^2} = 0 \tag{7}$$

The solution of Equation 7 is

$$h_x = C_1 x + C_2 \tag{8}$$

where h_x is the uplift head at the base of the top blanket at point x but measured upward from the top of the top blanket and C_1 and C_2 are constants. At $x = B$, $h_x = h_a$ and so

$$h_a = C_1 B + C_2 \qquad (9)$$

Also at $x = B$

$$-\frac{dh}{dx} = C_1 = -\frac{H - h_a}{\overline{X} + L_2 + B} \qquad (10)$$

where

H = the net head on the systems

$\overline{X}$ = the effective length of the riverside top blanket

L_2 = the base width of the levee

B = the berm width (landside levee toe to berm toe)

and so

$$C_2 = h_a + \frac{(H - h_a)B}{\overline{X} + L_2 + B} \qquad (11)$$

Thus Equation 8 may be expressed as

$$h_x = \frac{(H - h_a)(B - x)}{\overline{X} + L_2 + B} + h_a \qquad (12)$$

The uplift head at the landside levee toe h_t $(x = 0)$ is

$$h_t = \frac{(H - h_a)B}{\overline{X} + L_2 + B} + h_a \qquad (13)$$

Since the berm is impervious, there is no upward seepage in the berm and the underlying top blanket. Upward seepage occurs in the top blanket beyond the berm toe. The uplift safety factor for the berm only is

$$F = \frac{t_x \gamma_t'}{\gamma_w (h_x - t_x)} \qquad (14)$$

where

t_x = the berm thickness at point x

γ_t' = the buoyant unit weight of the berm

Equation 14 may be rewritten as

$$t_x = \frac{h_x}{1 + \frac{\gamma_t'}{\gamma_w F}} \tag{15}$$

6. If the berm is short, there will be seepage Q_{BT} up through the crack in the top blanket at the berm toe. From Equation 6

$$\frac{Q_{BT}}{k_f D} = \frac{H - h_a}{\underline{\overline{X}} + L_2 + B} - h_a c \tag{16}$$

If Q_{BT} is zero, then

$$H - h_a = h_a c(\underline{\overline{X}} + L_2 + B) \tag{17}$$

and the berm width B is

$$B = \frac{H - h_a}{h_a c} - (\underline{\overline{X}} + L_2) \tag{18}$$

7. The following is presented as an example:

$H = 30$ ft, $Z_b = 5$ ft, $D = 50$ ft, $\frac{k_f}{k_b} = 200$,

$c = \frac{1}{223.6}$ ft, $L_{LS} = 223.6$ ft, $\underline{\overline{X}} = L_2 = 418.5$ ft

$F_{LS} = F$	B , ft	
1.0	699.5	(See table, page 11,
1.5	1370.3	main report)

For $F_{LS} = 1.0$, $F = 1.5$, $h_a = 5$ ft

B , ft	h_t , ft	t , ft	$b = t_B$, ft	$Q_{BT}/k_f D$
699.5	20.64	12.30	3	0.0
600.0	19.73	11.84	3	0.00219
500.0	18.61	11.17	3	0.00486
400.0	17.22	10.33	3	0.00818

b = the berm thickness (berm toe) ; $x = B$.

Case II, Infinitely Pervious Berm

8. The permeability of the berm, both in the vertical and horizontal directions, is infinite. The berm offers no resistance to seepage flowing up through the top blanket. The berm acts only as a weighted filter to provide acceptable uplift safety factors. The basic second-order differential equation is

$$\frac{d^2h}{dx^2} = \frac{k_b h_x}{k_f D Z_b} = c^2 h \tag{19}$$

The solution is

$$h_x = C_1 e^{xc} + C_2 e^{-xc} \tag{20}$$

At $x = 0$, $h_x = h_t$ so that

$$h_t = C_1 + C_2 \tag{21}$$

At $x = B$, $h_x = h_a$

$$h_a = C_1 e^{Bc} + C_2 e^{-Bc} \tag{22}$$

The values of C_1 and C_2 are obtained by solving the set of simultaneous Equations 21 and 22:

$$C_1 = \frac{h_a - h_t e^{-Bc}}{2 \sinh (Bc)} \tag{23}$$

and

$$C_2 = \frac{h_t e^{Bc} - h_a}{2 \sinh (Bc)} \tag{24}$$

Equation 20 may be expressed as

$$h_x = \frac{h_a \sinh (xc) + h_t \sinh \left[(B - x)c\right]}{\sinh (Bc)} \tag{25}$$

The seepage gradient in the pervious foundation is

$$- \frac{dh}{dx} = \frac{c}{\sinh (Bc)} \left[h_t \cosh (B - x)c - h_a \cosh (xc) \right] \tag{26}$$

At $x = 0$

$$\left(\frac{H - h_t}{\underline{\overline{X}} + L_2} \right) = - \frac{dh}{dx} = \frac{c}{\sinh (Bc)} \left[h_t \cosh (Bc) - h_a \right] \tag{27}$$

which after algebraic manipulation becomes

$$h_t = \frac{H + h_a c \dfrac{\underline{\overline{X}} + L_2}{\sinh (Bc)}}{1 + \dfrac{c(\underline{\overline{X}} + L_2)}{\tanh (Bc)}} \tag{28}$$

At the berm toe $(x = B)$ the seepage flux per unit levee length is

$$\frac{Q_{BT}}{k_f D} = \frac{Q_B}{k_f D} - \frac{Q_{LS}}{k_f D} \tag{29}$$

$$\frac{Q_{BT}}{k_f D} = \frac{c}{\sinh (Bc)} \left(h_t - h_a \cosh Bc \right) - h_a c \tag{30}$$

If $Q_{BT} = 0$, then

$$h_t = h_a \left[\sinh (Bc) + \cosh (Bc) \right] \tag{31}$$

If Equations 28 and 31 are equated, one can obtain an expression for the berm width B :

$$B = \frac{1}{c} \ln \left\{ \frac{H}{h_a \left[1 + c(\underline{\overline{X}} + L_2) \right]} \right\} \tag{32}$$

9. The following example is presented:

$H = 30$ ft, $Z_b = 5$ ft, $D = 50$ ft, $\dfrac{k_f}{k_b} = 200$,

$$\overline{X} + L_2 = 418.5 \text{ ft}, \quad c = \frac{1}{223.6} \text{ ft}, \quad \gamma_w = \gamma_b' ,$$

$$B = 223.6 \ln \left[\frac{30}{\frac{5}{F_{LS}} \left(1 + \frac{418.5}{223.6} \right)} \right]$$

F_{LS}	B , ft
1.0	164.8
1.3	223.4
1.5	255.4

If B is greater than 165 ft; Q_{BT} is zero. If B is less than 165 ft, Q_{BT} is finite and the berm is short with $F_{LS} = 1.0$.

10. Another example is presented below with the data the same as above, plus

$$\gamma_w = \gamma_b' = \frac{\gamma_t'}{2} \quad (\gamma_t' \text{ is the moist weight of the berm})$$

$$B = 120 \text{ ft}, \quad F = 1.5 , \quad F_{LS} = 1.0$$

$$h_t = \frac{\left[30 + \frac{5 \times 418.5}{223.6 \sinh \left(\frac{120}{223.6} \right)} \right]}{\left[1 + \frac{418.5}{223.6} \times \frac{1}{\tanh \left(\frac{120}{223.6} \right)} \right]} = 9.68 \text{ ft}$$

$$\frac{Q_{BT}}{k_f D} = \frac{1}{223.6 \sinh \left(\frac{120}{223.6} \right)} \left[9.68 - 5 \cosh \left(\frac{120}{223.6} \right) \right] - \frac{1}{\sqrt{2000}}$$

$$= 0.00898$$

The berm thickness t_x is (from Equation 14 of the main text)

$$t_x = \left[\frac{h_x \, 1.5 - 5}{2} \right] = 0.75 \, h_x - 2.5$$

$$h_x = \frac{\left[5 \sinh \left(\frac{x}{223.6} \right) + 9.68 \sinh \left(\frac{120 - x}{223.6} \right) \right]}{\sinh \left(\frac{120}{223.6} \right)}$$

x , ft	h_x , ft	t_x , ft
0	9.68	4.76
30	8.30	3.73
60	7.08	2.81
90	5.99	1.99
120	5.00	1.25

Case III, Infinitely Pervious Berm in the Vertical Direction Only

11. The horizontal permeability of the berm is zero, while that for the vertical is infinite. Thus the berm permits water to be stored in it which produces a back hydraulic head on the seepage flowing up through the top blanket. The basic second-order differential equation is (See Equation 19 of the main report):

$$\frac{d^2h}{dx^2} = c^2(h_x - t_x) = h_x\theta + \xi \tag{33a}$$

where

$$\xi = \frac{c^2 Z_b \gamma_b'}{\gamma_w F + \gamma_t'} \tag{33b}$$

Setting

$$y = h_x\theta + \xi \tag{34}$$

permits expressing Equation 33a as

$$\frac{d^2y}{dx^2} = \theta y \tag{35}$$

where

$$\theta = \frac{c^2}{1 + \dfrac{\gamma_w F}{\gamma_c'}} \tag{36}$$

The solution of Equation 35 is

$$y = C_1 e^{x\sqrt{\theta}} + C_2 e^{-x\sqrt{\theta}} = \theta h_x + \xi \tag{37}$$

and so

$$h_x = \frac{C_1}{\theta} e^{x\sqrt{\theta}} + \frac{C_2 e^{-x\sqrt{\theta}}}{\theta} - \frac{\xi}{\theta} \tag{38}$$

where

$$\frac{\xi}{\theta} = \frac{Z_b \gamma_b'}{\gamma_t'} \tag{39}$$

At $x = B$, $h_x = h_a$

$$h_a = \frac{C_1}{\theta} e^{B\sqrt{\theta}} + \frac{C_2 e^{-B\sqrt{\theta}}}{\theta} - \frac{\xi}{\theta} \tag{40}$$

At $x = 0$, $h_x = h_t$

$$h_t = \frac{C_1}{\theta} + \frac{C_2}{\theta} - \frac{\xi}{\theta} \tag{41}$$

Solving the set of simultaneous Equations 40 and 41, one obtains

$$\frac{C_1}{\theta} = \frac{\left(h_a - h_t e^{-B\sqrt{\theta}}\right) + \frac{\xi}{\theta}\left(1 - e^{-B\sqrt{\theta}}\right)}{2 \sinh (B\sqrt{\theta})} \tag{42}$$

and

$$\frac{C_2}{\theta} = \frac{\left(h_t e^{B\sqrt{\theta}} - h_a\right) - \frac{\xi}{\theta}\left(1 - e^{B\sqrt{\theta}}\right)}{2 \sinh (B\sqrt{\theta})} \tag{43}$$

so that Equation 38 can be written as

$$h_x = \left(h_a + \frac{Z_b \gamma_b'}{\gamma_t}\right) \frac{\sinh (x\sqrt{\theta})}{\sinh (B\sqrt{\theta})} + \left(h_t + \frac{Z_b \gamma_b'}{\gamma_t}\right) \frac{\sinh \left[(B - x)\sqrt{\theta}\right]}{\sinh (B\sqrt{\theta})} - \frac{Z_b \gamma_b'}{\gamma_t'} \tag{44}$$

The seepage gradient in the pervious foundation is

$$- \frac{dh}{dx} = \left(h_t + \frac{Z_b \gamma_b'}{\gamma_t}\right) \frac{\sqrt{\theta} \cosh \left[(B - x)\sqrt{\theta}\right]}{\sinh (B\sqrt{\theta})} - \left(h_a + \frac{Z_b \gamma_b'}{\gamma_t}\right) \frac{\sqrt{\theta} \cosh (x\sqrt{\theta})}{\sinh (B\sqrt{\theta})} \tag{45}$$

At $x = 0$

$$-\frac{dh}{dx} = \frac{H - h_t}{\underline{X} + L_2} = \left(h_t + \frac{Z_b\gamma_b'}{\gamma_t'}\right)\frac{\sqrt{\theta}\cosh(B\sqrt{\theta})}{\sinh(B\sqrt{\theta})}$$

$$-\left(h_a + \frac{Z_b\gamma_b'}{\gamma_t'}\right)\frac{\sqrt{\theta}}{\sinh(B\sqrt{\theta})} \qquad (46)$$

from which one obtains

$$h_t = \frac{H + \dfrac{(\overline{\underline{X}} + L_2)\sqrt{\theta}}{\sinh(B\sqrt{\theta})}\left(h_a + \dfrac{Z_b\gamma_b'}{\gamma_t'}\right) - \dfrac{(\overline{\underline{X}} + L_2)\sqrt{\theta}}{\sinh(B\sqrt{\theta})}\dfrac{Z_b\gamma_b'}{\gamma_t'}\cosh(B\sqrt{\theta})}{1 + \dfrac{(\overline{\underline{X}} + L_2)\sqrt{\theta}}{\tanh(B\sqrt{\theta})}} \qquad (47)$$

The seepage flux up through the berm toe crack is

$$\frac{Q_{BT}}{k_f D} = \left(h_t + \frac{Z_b\gamma_b'}{\gamma_t'}\right)\frac{\sqrt{\theta}}{\sinh(B\sqrt{\theta})} - \left(h_a + \frac{Z_b\gamma_b'}{\gamma_t'}\right)\frac{\sqrt{\theta}}{\tanh(B\sqrt{\theta})} - h_a c \qquad (48)$$

If Q_{BT} is zero, then

$$h_t = h_a\left[\frac{c}{\sqrt{\theta}}\sinh(B\sqrt{\theta}) + \cosh(B\sqrt{\theta})\right] - \frac{Z_b\gamma_b'}{\gamma_t'}\left[\cosh(B\sqrt{\theta}) - 1\right] \qquad (49)$$

Equating Equations 47 and 49 permits the determination of the berm width B for the condition of no flow up through the berm toe crack. For short berms, B will be less than that calculated above. The equating of Equations 47 and 49 results in an implicit expression and the value of B must be found by trial. This can readily be done using a programmable hand calculator.

12. The uplift safety factor for the combined berm and top blanket is

$$F_c = \frac{Z_b \gamma' + t_x \gamma'_t}{(h_x - t_x)\gamma_w} \tag{50}$$

The berm thickness t_x at point x is

$$t_x = \frac{h_x - \dfrac{Z_b \gamma'_b}{\gamma'_t}}{1 + \dfrac{\gamma'_t}{\gamma_w F_c}} \tag{51}$$

13. An example is worked out as follows:

$H = 30$ ft, $Z_b = 5$ ft, $D = 50$ ft, $\frac{k_f}{k_b} = 200$,

$\overline{X} + L_2 = 418.5$ ft, $\frac{1}{c} = 223.6$ ft,

$\gamma_w = \gamma'_b = \gamma'_t = 62.4$ lb/cu ft

For $Q_{BT} = 0$

F_c	B , ft $F_{LS} = 1.0$	$F_{LS} = 1.3$	$F_{LS} = 1.5$	
1.0	182	--	--	(See tabulation, page 22, main report)
1.3	197	256	--	
1.5	207	--	304	

14. The following example is for a short berm with $Q_{BT} \neq 0$ and the same data as above except that $F_{LS} = 1.0$, $F_c = 1.5$, and $B = 150$ ft:

$B\sqrt{\theta} = 0.4242769669$

$\sinh (B\sqrt{\theta}) = 0.4371211108$

$\cosh (B\sqrt{\theta}) = 1.091363764$

$$h_t = \frac{30 + \dfrac{418.5 \times 10}{223.6\sqrt{2.5}\sinh (B\sqrt{\theta})} - \dfrac{418.5 \times 5 \times \cosh (B\sqrt{\theta})}{223.6\sqrt{2.5}\sinh (B\sqrt{\theta})}}{1 + \dfrac{418.5 \cosh (B\sqrt{\theta})}{223.6\sqrt{2.5}\sinh (B\sqrt{\theta})}}$$

$= 10.550$ ft

x, ft	t_x, ft
0	3.33
50	2.07
100	0.96
150	0.00

Case IV, Permeability of Seepage Berm Equal to That of the Top Semipervious Blanket

15. This is a limiting condition for Case V and will not be discussed.

Case V, Semipervious Berm

16. The permeability ratio of the berm to the top semipervious blanket $\bar{K}$ ranges between zero and unity. The basic second-order differential equation is

$$\frac{d^2h}{dx^2} = \frac{k_t\gamma_t'}{k_f D\gamma_w F} = \frac{k_t}{k_b}\frac{k_b Z_b\gamma_t'}{k_f D Z_b\gamma_w F} = \frac{\bar{K}c^2 Z_b\gamma_t'}{\gamma_w F} = \theta \tag{52}$$

The solution is

$$h_x = \frac{\theta x^2}{2} + C_1 x + C_2 \tag{53}$$

At $x = 0$, $h_x = h_t = C_2$, and at $x = B$, $h_x = h_a$, so that

$$h_a = \frac{\theta B^2}{2} + C_1 B + h_t \tag{54}$$

$$C_1 = -\frac{h_t - h_a}{B} - \frac{\theta B}{2} \tag{55}$$

Equation 53 may be expressed as

$$h_x = \frac{\theta x^2}{2} - (h_t - h_a)\frac{x}{B} - \frac{\theta Bx}{2} + h_t \tag{56}$$

The seepage gradient in the pervious foundation is

$$- \frac{dh}{dx} = -\theta x + \frac{h_t - h_a}{B} - \frac{\theta B}{2} \tag{57}$$

At $x = 0$

$$- \frac{dh}{dx} = \frac{H - h_t}{\underline{X} + L_2} = \frac{h_t - h_a}{B} + \frac{\theta B}{2} \tag{58}$$

so that

$$h_t = \frac{H + \dfrac{h_a(\overline{X} + L_2)}{B} - \dfrac{\theta B(\overline{X} + L_2)}{2}}{1 + \left(\dfrac{\overline{X} + L_2}{B}\right)} \tag{59}$$

The seepage flux up through the berm toe crack is

$$\begin{aligned} \frac{Q_{BT}}{k_f D} &= - \theta B + \frac{h_t - h_a}{B} + \frac{\theta B}{2} - h_a c \\ &= \frac{h_t - h_a}{B} - \frac{\theta B}{2} - h_a c \end{aligned} \tag{60}$$

If Q_{BT} is zero, then

$$\begin{aligned} h_t &= \frac{\theta B^2}{2} + h_a \ (1 + Bc) \\ &= \frac{\overline{K} c^2 z_b \gamma_t' B^2}{2\gamma_w F} + h_a \ (1 + Bc) \end{aligned} \tag{61}$$

Equations 59 and 61 are equated so that

$$B = \alpha\left(-1 + \sqrt{1 + \frac{\Delta}{\alpha^2}}\right) \tag{62}$$

where

$$\alpha = (\overline{X} + L_2) + \frac{h_a c}{\theta} \tag{63}$$

and

$$\Delta = \frac{2}{\theta} \left\{ H - h_a \left[1 + c(\underline{\overline{X}} + L_2) \right] \right\} \tag{64}$$

The expression for the berm thickness t_x, using Equation 84 of the main report, is

$$t_x = \frac{h_x - \dfrac{Z_b \gamma_t' \overline{K}}{\gamma_w F}}{1 + \dfrac{\gamma_t'}{\gamma_w F}} \tag{65}$$

17. An example of this is computed as follows:

$H = 30$ ft, $Z_b = 5$ ft, $D = 50$ ft, $\underline{\overline{X}} + L_2 = 418.5$ ft,

$\frac{1}{c} = 223.6$ ft, $\gamma_w = \gamma_b' = \gamma_t' = 62.4$ lb/cu ft, $Q_{BT} = 0$,

$\frac{k_f}{k_b} = 200$

	B , ft				
		F = 1.5			
$\overline{K}$	$F = F_{LS} = 1.0$	$F_{LS} = 1.0$	$F_{LS} = 1.2$	$F_{LS} = 1.5$	
1.0	209.4	264.7	316.0	370.4	(See table, page 31, of the main report)
1×10^{-1}	535.3	577.7	753.5	975.2	
1×10^{-2}	676.7	648.1	938.1	1307.7	
1×10^{-3}	697.2	698.0	964.8	1363.7	
1×10^{-4}	699.3	699.4	967.6	1369.7	
1×10^{-5}	699.5	699.5	967.9	1370.3	
1×10^{-6}	699.6	699.6	967.9	1370.4	
1×10^{-7}	699.6	699.6	967.9	1370.4	

The values of B are sensitive to variations of $\overline{K}$ and F_{LS} .

18. An example of a short berm follows. The same data as above is used, plus

$F = 1.5$, $F_{LS} = 1.0$, $B = 300$ ft, $\overline{K} = 0.1$

$$h_t = \frac{30 + \frac{5 \times 418.5}{300} - \frac{0.1 \times 5 \times 300 \times 418.5}{1.5 \times 50,000}}{1 + \frac{418.5}{300}} = 15.089 \text{ ft}$$

$$h_x = \frac{0.1 \times 5}{50,000 \times 1.5 \times 2} (x^2 - 300x) - \left(\frac{15.089 - 5}{300}\right) x + 15.089$$

$$t_x = \frac{h_x - \frac{5 \times 0.1}{1.5}}{1 + \frac{1}{1.5}} = 0.6h_x - 0.2$$

x, ft	h_x, ft	t_x, ft
0	15.089	8.85
50	13.37	7.82
100	11.66	6.80
150	9.97	5.78
200	8.30	4.78
250	6.64	3.78
300	5.00	2.80

The seepage flux up through the top blanket crack at the berm toe, per unit berm length, is

$$\frac{Q_{BT}}{k_f D} = - \frac{0.1 \times 5 \times 300}{50,000 \times 1.5 \times 2} + \frac{15.089 - 5}{300} - \frac{5}{223.6} = 0.01027$$

Case VI, Variable Uplift Safety Factors

19. The uplift factor for the berm only varies in a linear manner from a minimum of F_B at the berm toe ($x = B$) to a maximum of F_o at the levee landside toe ($x = 0$). The uplift safety factor for the top blanket at the berm toe is F_{LS}. The permeability ratio $\bar{K}$ varies from zero to unity. The case for $\bar{K}$ greater than unity will be discussed under Case VII. The basic second-order differential equation is Equation 87 of the main report. If $F_B \neq F_{LS}$ then Equation 105 of the main report must be modified as follows:

$$B^2\left[\frac{F_o}{(F_o - F_B)^2}\ln\left(\frac{F_o}{F_B}\right) - \frac{1}{F_o - F_B}\right] + B\left[\frac{\overline{X} + L_2}{F_o - F_B}\ln\left(\frac{F_o}{F_B}\right) + \frac{\gamma'_b}{F_{LS}\overline{K}c\gamma'_t}\right]$$

$$+\left[\frac{\gamma'_b}{\gamma'_t}\frac{\left(\overline{X} + L_2 + L_{LS}\right)}{F_{LS}\overline{K}c} - \frac{H\gamma_w}{Z_b\overline{K}\gamma'_t c^2}\right] = 0 \tag{66}$$

Set

$$\overline{A} = \left[\frac{F_o}{(F_o - F_B)^2}\ln\left(\frac{F_o}{F_B}\right) - \frac{1}{F_o - F_B}\right] \tag{67}$$

$$\overline{C} = \left[\frac{\overline{X} + L_2}{F_o - F_B}\ln\left(\frac{F_o}{F_B}\right) + \frac{\gamma'_b}{F_{LS}\overline{K}c\gamma'_t}\right] \tag{68}$$

$$\overline{D} = \left[\frac{\gamma'_b}{\gamma'_t}\frac{\left(\overline{X} + L_2 + L_{LS}\right)}{F_{LS}\overline{K}c} - \frac{H\gamma_w}{Z_b\overline{K}\gamma'_t c^2}\right] \tag{69}$$

The berm width B , for $Q_{BT} = 0$, is

$$B = \frac{\overline{C}}{2\overline{A}}\left(-1 + \sqrt{1 - \frac{4\overline{A}\overline{D}}{\overline{C}^2}}\right) \tag{70}$$

The uplift head h_x and the berm thickness t_x are given by Equations 99 and 110 of the main report.

20. An example is presented as follows:

$H = 30$ ft, $Z_b = 5$ ft, $D = 50$ ft, $\overline{X} + L_2 = 418.5$ ft,

$\frac{1}{c} = L_{LS} = 223.6$ ft, $\gamma_w = \gamma'_b = \gamma'_t = 62.4$ lb/cu ft

$\frac{k_f}{k_b} = 200$, $F_o = 1.5$, $\overline{K} = 0.1$

F_{LS}	F_B	B , ft
1.0	1.0	555.5
1.0	1.1	560.5
1.0	1.2	565.6
1.0	1.3	570.0
1.0	1.4	574.0

$F_B = F_{LS}$	B , ft
1.0	555.5
1.1	646.7
1.2	733.8
1.3	817.5
1.4	897.8
1.49	967.6
1.499	974.5
1.4999	975.2
1.49999	975.2

The above results should be compared with those tabulated for Case V for $\bar{K} = 0.1$. If F_o , F_{LS} , and $\bar{K}$ are fixed, B varies only slightly as F_B varies. If $F_B = F_{LS}$, B varies in a significant manner as F_B varies.

21. For a short berm, B is less than that given by Equation 70. The uplift head at the base of the top blanket is

$$h_x = \theta(\phi \ln \phi - \phi) + C_1\phi + C_2 \tag{71}$$

where

$$\theta = \frac{\bar{K}B^2 Z_b c^2 \gamma_b'}{\gamma_w(F_o - F_B)^2} \tag{72}$$

and

$$\phi = F_o - (F_o - F_B)\frac{x}{B} \tag{73}$$

At $x = B$, $h_x = h_a$, $\phi = F_B$

$$h_a = \theta(F_B \ln F_B - F_B) + C_1F_B + C_2 \tag{74}$$

At $x = 0$, $h_x = h_t$, $\phi = F_o$

$$h_t = \theta(F_o \ln F_o - F_o) + C_1 F_o + C_2 \tag{75}$$

Subtracting Equation 75 from Equation 73, and then with minor algebraic manipulation one has

$$C_1 = \frac{h_t - h_a}{F_o - F_B} - \frac{\theta}{F_o - F_B}\left[F_o \ln F_o - F_B \ln F_B - (F_o - F_B)\right] \tag{76}$$

and from Equation 74

$$C_2 = h_a - \theta(F_B \ln F_B - F_B) - C_1 F_B \tag{77}$$

From Equation 110 of the main report, the berm thickness at point x is

$$t_x = \frac{h_x - \dfrac{\gamma_t' Z_b \bar{K}}{\gamma_w\left[F_o - (F_o - F_B)\dfrac{x}{B}\right]}}{\left(1 + \dfrac{\gamma_t'}{\gamma_w}\right)\dfrac{1}{F_o - (F_o - F_B)\dfrac{x}{B}}} \tag{78}$$

The value of h_t at $x = 0$ is

$$h_t = \frac{H - (\bar{X} + L_2)(F_o - F_B)\frac{\theta}{B}\ln F_o + (\bar{X} + L_2)\frac{\theta}{B}(F_o \ln F_o - F_B \ln F_B - F_o + F_B) + \dfrac{(\bar{X} + L_2)}{B} h_a}{1 + \dfrac{(\bar{X} + L_2)}{B}} \tag{79}$$

The seepage flux per unit levee length up through the top blanket crack at the berm toe is

$$\frac{Q_{BT}}{k_f D} = \frac{F_o - F_B}{B}(\theta \ln F_B + C_1) - h_a c \tag{80}$$

22. An example is presented as follows:

$$H = 30 \text{ ft}, \quad Z_b = 5 \text{ ft}, \quad D = 50 \text{ ft}, \quad \frac{k_f}{k_b} = 200 \ , \quad \overline{K} = 0.1 \ ,$$

$$\underline{\overline{X}} + L_2 = 418.5 \text{ ft} \ , \quad \frac{1}{c} = L_{LS} = 223.6 \text{ ft} \ ,$$

$$\gamma_w = \gamma_b' = \gamma_t' = 62.4 \text{ lb/cu ft}, \quad B = 500 \text{ ft}, \quad h_a = 5 \text{ ft},$$

$$F_o = 1.5 \ , \quad F_B = 1.1 \ , \quad F_{LS} = 1.0$$

$$\theta = \frac{0.1 \times 500^2 \times 5 \times 62.4}{62.4(1.5 - 1.1)} = 15.625$$

$$h_t = \frac{\left[30 - (418.5)(1.5 - 1.1)\frac{15.625}{500}\ln 1.5 + (418.5)\frac{15.625}{500}(1.5 \ln 1.5 - 1.1 \ln 1.1 - 1.5 + 1.1) - \frac{418.5 \times 5}{500}\right]}{1 + \frac{418.5}{500}}$$

$$= 13.633979^*$$

$$C_1 = \left[\frac{(13.633979 - 5)}{(1.5 - 1.1)} - \frac{15.625}{(1.5 - 1.1)}(1.5 \ln 1.5\right.$$

$$\left. - 1.1 \ln 1.1 - 1.5 + 1.1)\right] = 17.54758561^*$$

$$C_2 = 5 - 15.625(1.1 \ln 1.1 - 1.1) - 17.54758561 \times 1.1$$

$$= 1.247012114^*$$

$$\phi = 1.5 - (1.5 - 1.1)\frac{x}{500} = 1.5 - 0.0008x$$

$$h_x = 15.625(\phi \ln \phi - \phi) + 17.54758561\phi + 1.247012114$$

$$t_x = \frac{h_x - \dfrac{62.4 \times 5 \times 0.1}{62.4\left[1.5 - (1.5 - 1.1)\right]\dfrac{x}{500}}}{\left(1 + \dfrac{62.4}{62.4}\right)\left[\dfrac{1}{1.5 - (1.5 - 1.1)\dfrac{x}{500}}\right]} = \frac{h_x - \dfrac{0.5}{1.5 - 0.0008x}}{\dfrac{2}{1.5 - 0.0008x}}$$

with the following results tabulated:

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* Values in hand computer program.

x, ft	ϕ	h_x, ft	t_x, ft
0	1.50	13.634	9.98
100	1.42	11.757	8.10
200	1.34	9.951	6.42
300	1.26	8.219	4.93
400	1.18	6.567	3.62
500	1.10	5.000	2.50

Case VII, Pervious Berm

23. The basic second-order differential equation has not been solved and therefore it is necessary to use a finite difference method of solution for the uplift head h_x. The method is shown on page 46 of the main report. An expression for the intial value of h_{2_i} is given on the lower middle part of page 47 of the main report, which contains h_a. The definition of h_a is not general and the following modification is necessary:

$$h_{2_i} = \frac{Z_b \gamma_b'}{\gamma_w F_{LS}} \left[\frac{\Delta x}{L_{LS}} + 1 - \frac{c^2 \Delta x^2 (\gamma_t' + \gamma_w F_{LS})}{2\left(\frac{F\gamma_b'}{F_{LS}} + \gamma_w F\overline{K} + \gamma_t'\overline{K} - \gamma_b'\right)} \right] \tag{81}$$

24. The solution for the short berm case cannot be solved in the manner noted above because of the seepage Q_{BT}. One must go to the levee landside toe and proceed stepwise to the berm toe. An initial value of h_t is assumed and after a series of stepwise computations, a value of h_a is calculated. This value will generally not be that of Equation 2. A new value of h_t is assumed and the process is repeated. This procedure is continued until the calculated value of h_a is sufficiently close to the value of h_a from Equation 2. The initial value of h_{2_i} is found using

$$h_{2_i} = h_t\left(\frac{\Delta x}{\underline{\overline{X}} + L_2} + 1\right) - \frac{H\Delta x}{\underline{\overline{X}} + L_2} + \left[\frac{\frac{c^2 \Delta x^2}{2}\left(h_t + \frac{Z_b \gamma_b'}{\gamma_t'}\right)}{\frac{\gamma_w F h_t}{\gamma_t' Z_b \overline{K}} + \frac{\gamma_w F}{\gamma_t'} + 1 - \frac{\gamma_b'}{\gamma_t \overline{K}}} \right] \tag{82}$$

Knowing h_t, which is also h_{3_i}, and h_{2_i}, one can compute h_{1_i} by using

$$h_1 = 2h_2 - h_3 + \frac{\Delta x^2 c^2 \left(h_2 + \frac{Z_b \gamma'_b}{\gamma'_t}\right)}{\frac{\gamma_w F h_2}{\gamma'_t Z_b \overline{K}} + \frac{\gamma_w F}{\gamma'_t} + 1 - \frac{\gamma'_b}{\gamma'_t \overline{K}}} \tag{83}$$

The values of the berm thickness at intervals of Δx are computed using

$$t_x = \frac{h_1 \gamma_w F - Z_b \gamma'_b}{\gamma_w F + \gamma_t} \tag{84}$$

In some cases the uplift safety factor F may not be a constant but a function of x. For the latter case the value of F will vary for each step. The flux $Q_{BT}/k_f D$ is

$$\frac{Q_{BT}}{k_f D} = \frac{h_2 - h_a}{\Delta x} - h_a c \tag{85}$$

25. An example is presented as follows:

$H = 30$ ft, $Z_b = 5$ ft, $D = 50$ ft, $\frac{k_f}{k_b} = 200$, $\overline{K} = 10$,

$\overline{X} + L_2 = 418.5$ ft, $\frac{1}{c} = 223.6$ ft $= L_{LS}$, $B = 150$ ft,

$h_a = 1.5$, $\gamma_w = \gamma'_b = \gamma'_t = 62.4$ lb/cu ft, $F_{LS} = 1.0$,

$F = 1.5$, $\Delta x = 10$ ft

$$h_{2_i} = h_t\left(\frac{418.5 + 10}{418.5}\right) - \frac{300}{418.5} + \frac{0.001(h_t + 5)}{0.03h_t + 2.4}$$

$$h_1 = 2h_2 - h_3 + \frac{0.002(h_2 + 5)}{0.03h_t + 2.4}$$

$$t_x = \frac{h_1 \times 1.5 - 5}{1.5 + 1} = 0.6h_1 - 2$$

Estimate $h_{t_i} = 13$ ft

h_t , ft	h_a , ft, Calculated
13.0	5.32
12.8	5.03
12.7	4.88
12.78	5.00*

* Agrees with given value of 5.0 ft.

Use $h_t = 12.78$ ft, $\Delta x = 10$ ft, $B = 150$ ft to give

x , ft	h_x , ft	t_x , ft
0	12.78	5.67
10	11.98	5.19
20	11.56	4.93
30	11.12	4.67
40	10.68	4.41
50	10.22	4.13
60	9.75	3.85
70	9.27	3.56
80	8.78	3.27
90	8.28	2.97
100	7.77	2.66
110	7.24	2.34
120	6.70	2.02
130	6.15	1.69
140	5.58	1.35
150	5.00	1.00*

* If $F = F_{LS}$ then for $\bar{K} > 1.0$, $b = t_B = 0$.
If $F > F_{LS}$ then for $\bar{K} > 1.0$, $b = t_B > 0$.

$$\frac{Q_{BT}}{k_f D} = \frac{5.58 - 5.00}{10} - \frac{5}{223.6} = 0.0356$$

Case VIII, Berm with Constant Slope

26. The solution for this case is given in Supplement No. 1 for

the condition that Q_{BT} , the seepage up through the top blanket crack at the berm toe, is zero. The definition of h_a requires revision to that of Equation 2 of this supplement to render the solution more general. The thickness of the berm t_x at point x is

$$t_x = t - (t - b) \frac{x}{B} \tag{86}$$

The thickness of the berm toe b at $x = B$ is

$$b = \frac{Z_b\left(\frac{\gamma_b' Z_b}{\gamma_t' F_{LS}} - \overline{K}\right)}{1 + \frac{\gamma_w F_B}{\gamma_t'}} \tag{87}$$

for $\overline{K}$ less than 1.0. If

$$\frac{\gamma_b' F_B}{\gamma_t' F_{LS}} \leq \overline{K} \tag{88}$$

then b is zero. The expression for y is revised (coordinate change) to

$$h_x - t + (t - b) \frac{x}{B} = y \tag{89}$$

The expression for ξ is also revised for the same reason:

$$\xi = \overline{K} + \frac{t_x}{Z_b} = \overline{K} + \frac{t}{Z_b} - \frac{(t - b)x}{BZ_b} \tag{90}$$

The basic second-order differential equation is

$$\frac{d^2y}{dx^2} = \frac{\psi y}{\xi} \tag{91}$$

where

$$\psi = \left(\frac{Z_b Bc}{t - b}\right)^2 \overline{K} \tag{92}$$

The solutions for y and $dy/d\xi$ are

$$y = h_x - t_x = \omega_x[C_1 I_1(\omega_x) + C_2 K_1(\omega_x)] \tag{93}$$

$$\frac{dy}{d\xi} = 2\Psi[C_1 I_o(\omega_x) - C_2 K_o(\omega_x)] \tag{94}$$

where

$$\omega_x = 2\sqrt{\psi\xi} \tag{95}$$

The values of C_1 and C_2 are those of Supplement No. 1:

$$C_1 = (h_a - b)K_o(\omega_B) + \left[\left(h_a - \frac{t - b}{Bc}\right)\sqrt{1 + \frac{b}{Z_b\overline{K}}}\right]K_1(\omega_B) \tag{96}$$

and

$$C_2 = (h_a - b)I_o(\omega_B) - \left[\left(h_a - \frac{t - b}{Bc}\right)\sqrt{1 + \frac{b}{Z_b\overline{K}}}\right]I_1(\omega_B) \tag{97}$$

where

$$\omega_B = \frac{2Z_b Bc\overline{K}}{t - b}\sqrt{1 + \frac{b}{Z_b\overline{K}}} \tag{98}$$

From Equation 37 of Supplement No. 1,

$$H = h_t + (\underline{\overline{X}} + L_2)\left\{\frac{t - b}{B} + \frac{2Z_b Bc^2\overline{K}}{t - b}\left[C_1 I_o(\omega_o) - C_2 K_o(\omega_o)\right]\right\} \tag{99}$$

where

$$\omega_o = \frac{2Z_b Bc\bar{K}}{t - b} \sqrt{1 + \frac{t}{Z_b \bar{K}}} \tag{100}$$

As noted in Supplement No. 1, values of the berm width B and the berm thickness t at $x = 0$ are assumed to calculate H. These values are varied by trial until the calculated value of H agrees with the given value.

27. The uplift safety factor at point x, F_x, is not a constant but varies with a maximum between $x = 0$ and $x = B$. For the combined berm and top blanket,

$$F_x(\text{combined}) = \frac{Z_b \gamma_b' + t_x \gamma_t'}{(h_x - t_x)\gamma_w} \tag{101}$$

and for the berm only

$$F_x(\text{berm}) = \frac{(Z_b \bar{K} + t_x)\gamma_t'}{(h_x - t_x)\gamma_w} \tag{102}$$

The lower value will control.

28. The basic second-order differential equation for the short berm is given by Equation 91 and the solutions are those of Equations 93 and 94. The evaluations of constants C_1 and C_2 are changed because Q_{BT} is not zero. At $x = 0$, $h_x = h_t$ and $y = h_t - t$,

$$\frac{dh}{dx} = -\left(\frac{H - h_t}{\bar{X} + L_2}\right) \tag{103}$$

and

$$\omega_o = \frac{2Z_b Bc\bar{K}}{t - b} \sqrt{1 + \frac{t}{Z_b \bar{K}}} \tag{104}$$

Equation 93 becomes

$$h_t - t = \omega_o[C_1 I_1(\omega_o) + C_2 K_1(\omega_o)] \tag{105}$$

Equation 94 becomes

$$\frac{dy}{d\xi} = \frac{dy}{dx} \cdot \frac{dx}{d\xi} = \left(\frac{dh}{dx} + \frac{t - b}{B}\right)\left(\frac{-BZ_b}{t - b}\right) = \left[-\left(\frac{H - h_t}{\overline{X} + L_2}\right) + \frac{t - b}{B}\right]\left(\frac{-BZ_b}{t - b}\right)$$

$$= \frac{(H - h_t)}{(\overline{X} + L_2)}\frac{BZ_b}{(t - b)} - Z_b = Z_b\left[\frac{(H - h_t)}{(\overline{X} + L_2)}\frac{B}{(t - b)} - 1\right]$$

$$= 2\left(\frac{Z_b Bc}{t - b}\right)^2 \overline{K}\left[C_1 I_o(\omega_o) - C_2 K_o(\omega_o)\right] \qquad (106)$$

Solving the simultaneous Equations 105 and 106, one obtains

$$C_1 = (h_t - t)K_o(\omega_o) + \left[\left(\frac{H - h_t}{\overline{X} + L_2} - 1\right)\left(\frac{t - b}{Bc}\right)\sqrt{1 + \frac{t}{Z_b\overline{K}}}\right]K_1(\omega_o) \qquad (107)$$

$$C_2 = (h_t - t)I_o(\omega_o) - \left[\left(\frac{H - h_t}{\overline{X} + L_2} - 1\right)\left(\frac{t - b}{Bc}\right)\sqrt{1 + \frac{t}{Z_b\overline{K}}}\right]I_1(\omega_o) \qquad (108)$$

At $x = 0$, the uplift safety factor is

$$F_o(\text{combined}) = \frac{(Z_b\gamma_b' + t\gamma_t')}{(h_t - t)\gamma_w} = F_{o_c} \qquad (109)$$

and

$$F_o(\text{berm only}) = \frac{(Z_b\overline{K} + t)\gamma_t'}{(h_t - t)\gamma_w} = F_{o_B} \qquad (110)$$

From which

$$h_t = \frac{Z_b\gamma_b' + t\left(\gamma_t' + \gamma_w F_{o_c}\right)}{F_{o_c}\gamma_w} \qquad (111)$$

$$h_t = \frac{Z_b \gamma_t \overline{K} + t\left(\gamma_t' + \gamma_w F_{o_B}\right)}{F_{o_B} \gamma_w} \qquad (112)$$

One must assume values of t and h_t for use in calculating C_1 and C_2. The value of $(h_t - t)$ can be readily calculated:

$$(h_t - t) = \frac{Z_b \gamma_b' + t\gamma_t'}{F_{o_c} \gamma_w} \qquad (113)$$

or

$$(h_t - t) = \frac{(Z_b \overline{K} + t)\gamma_t'}{F_{o_B} \gamma_w} \qquad (114)$$

using F_{o_c} or F_{o_B}, whichever is controlling. Also

$$(H - h_t) = H - (h_t - t) - t \qquad (115)$$

Values of t are assumed and C_1 and C_2 found. The correct value of t is that which gives a calculated value of $h_a - b$ equal to that given, using

$$(h_a - b) = \omega_B [C_1 I_1(\omega_B) + C_2 K_1(\omega_B)] \qquad (116)$$

where Equation 98 gives ω_B.

29. An example is presented as follows:

$H = 30$ ft, $Z_b = 5$ ft, $D = 50$ ft, $\frac{k_f}{k_b} = 200$,

$\underline{X} + L_2 = 418.5$ ft, $\frac{1}{c} = 223.6$ ft, $h_a = 5$ ft, $B = 500$,

$\gamma_w = \gamma_b' = \gamma_t' = 62.4$ lb/cu ft, $F_{LS} = 1.0$,

$F_o = F_B = 1.5$, $\overline{K} = 0.1$

$$b = \frac{5\left(\frac{1.5 \times 62.4}{1 \times 62.4} - 0.1\right)}{1 + \frac{62.4}{62.4} \times 1.5} = 2.8 \text{ ft}$$

$$\left(h_a - b\right) = 5 - 2.8 = 2.2 \text{ ft}$$

$$\left(h_t - t\right) = \frac{(5 \times 0.1 + t)62.4}{1.5 \times 62.4} = \frac{0.5 + t}{1.5}$$

$$\omega_B = \frac{2 \times 5 \times 500 \times 0.1}{223.6(t - 2.8)} \sqrt{1 + 2 \times 2.8} = \frac{500\sqrt{6.6}}{223.6(t - 2.8)}$$

$$\omega_o = \frac{2 \times 5 \times 500 \times 0.1}{223.6(t - 2.8)} \sqrt{1 + 2t} = \frac{500\sqrt{1 + 2t}}{223.6(t - 2.8)}$$

$$\omega_x = \frac{2 \times 5 \times 500 \times 0.1}{223.6(t - 2.8)} \sqrt{1 + 2t - \frac{(t - 2.8)x}{500 \times 5 \times 0.1}}$$

For various values of t, the following results occur:

t , ft	$\left(h_t - t\right)$, ft	ω_o	[ ]*	$I_o(\omega_o)$	$I_1(\omega_o)$	$K_o(\omega_o)$	$K_1(\omega_o)$
10.6	7.40	1.350766	13.77372	1.5109	0.8479	0.2600	0.3451
10.7	7.46667	1.339666	13.20252	1.5016	0.8318	0.2639	0.3510
10.8	7.5333	1.32881	12.62540	1.4926	0.8223	0.2677	0.3567

t , ft	C_1	C_2	ω_B	$I_1(\omega_B)$	$K_1(\omega_B)$	$\left(h_a - b\right)$, ft , Calculated
10.6	6.6773	-0.4981	0.73650	0.3938	0.9755	1.5788
10.7	6.6045	+0.2301	0.72718	0.3882	0.9939	2.0307
10.8	6.5202	+0.8624	0.71809	0.3827	1.0123	2.4187

By interpolation $t = 10.744$ ft

$$\omega_x = \frac{500}{223.6(10.744 - 2.8)} \sqrt{1 + 2 \times 10.744 - \frac{(10.744 - 2.8)x}{500 \times 5 \times 0.1}}$$

$$t_x = 10.744 - \frac{(10.744 - 2.8)x}{500}$$

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* Coefficient of $K_1(\omega_o)$ and $I_1(\omega_o)$ of Equations 107 and 108, respectively.

$$(h_t - t) = \frac{0.5 + 10.744}{1.5} = 7.496 \text{ ft}$$

$$h_t = 7.496 + 10.744 = 18.240$$

$$C_1 = 7.496K_o(1.334856) + \left[\left(\frac{30 - 18.24}{418.5} - \frac{10.744 - 2.8}{500}\right) 223.6\sqrt{1 + 2 \times 10.744}\right] K_1(1.334856)$$

$$C_2 = 7.496I_o(1.334856 + [\,] I_1(1.334856)$$

$$C_1 = 6.5685$$

$$C_2 = 0.51066$$

$$(h_x - t_x) = \omega_x[C_1 I_1(\omega_x) + C_2 K_1(\omega_x)]$$

$$F_{x_B} = \frac{0.5 + t_x}{h_x - t_x}$$

x , ft	ω_x	$I_1(\omega_x)$	$K_1(\omega_x)$	t_x , ft	$h_x - t_x$, ft	F_{x_B}
0	1.334856	0.8276	0.3535	10.744	7.497	1.50
100	1.23696	0.7445	0.4103	9.1552	6.308	1.53
200	1.13061	0.6606	0.4851	7.5664	5.186	1.56
300	1.01317	0.5745	0.5887	5.9776	4.128	1.57
400	0.88019	0.4841	0.7425	4.3888	3.133	1.56
500	0.72315	0.3857	1.0020	2.8000	2.202	1.50

The solution for a short berm for this case is very sensitive to the value of t .

Conclusions

30. The conclusions of the main report and Supplements No. 1 and 2 remain unchanged. Solutions are presented in this supplement for a coordinate system having an origin at the levee landside toe and that is positive landward. The solutions are more general by having the uplift safety factor for the top blanket at the berm toe different than that for either the berm or the combined berm and top blanket at the berm toe.

31. Solutions are also presented for berms which are shorter than regular berms. The uplift safety factor for the top blanket F_{LS} at the berm toe is assumed to be unity and a crack occurs in the top blanket at the berm toe parallel to the berm toe. The seepage up through the crack Q_{BT} is assumed to occur. While piping at this crack is not precluded, it is assumed that for a given flood no serious damage will occur. For such berms, it may be desirable to install a trench drain along the berm toe.

32. Some of the solutions are explicit and direct; others are implicit and require a hunt-and-seek technique to obtain answers. Some solutions are very sensitive to variations of the variables and require a fair number of significant places in the numerical values used. The final results are given to three figures, which may be too many figures. Considering the simplifying assumptions used, the solutions presented are highly approximate to real situations and should be used only as a guide to the designer's judgement. The solutions given are deterministic. It is suggested that a probabilistic approach be used in design.

Appendix A: Notation

$\overline{A}$	A constant (see Equation 67)
b	Thickness of berm at the landside toe
B	Seepage berm width from the landside levee toe to the landside berm toe
c	$(k_b/k_f DZ_b)^{1/2}$
$C_1, C_2, C_3, C_4, C_5, C_6$	Constants
$\overline{C}$	A constant (see Equation 68)
D	Pervious foundation thickness
$\overline{D}$	A constant (see Equation 69)
e	2.71828
F_B	Uplift safety factor at landside berm toe
F_{LS}	Uplift safety factor for top semipervious blanket at berm toe
F_o	Uplift safety factor at landside levee toe
F_{o_B}	Uplift safety factor at levee landside toe for berm only (x = 0)
F_{o_c}	Uplift safety factor at levee landside toe for combined berm and top blanket (x = 0)
F_x	Uplift safety factor at point x
h	Seepage uplift head at base of top blanket at point x, referenced to top of top blanket
h_a	Allowable seepage uplift head at the landside berm toe (measured at base of top blanket referenced to top of top blanket)
H	Net hydaulic head between river flood level and the landside upper surface of the top blanket
i	Seepage gradient

$I_o(\ )$ Modified Bessel function, first kind, zero order

$I_1(\ )$ Modified Bessel function, first kind, first order

k_b Vertical permeability coefficient of top blanket

k_f Horizontal coefficient of permeability of the previous foundation

k_t Vertical permeability coefficient of berm

$\bar{K}$ Permeability ratio of berm to top blanket, equals k_t/k_b

$K_o(\ )$ Modified Bessel function, second kind, zero order

$K_1(\ )$ Modified Bessel function, second kind, first order

L_1 Width of riverside top blanket measured from riverside levee toe to riverbank

L_2 Base width of levee

L_{LS} Effective length of landside top blanket measured landward from berm toe

Q_B Seepage per unit length of levee in the pervious foundation at berm toe (x = B)

Q_{BT} Seepage per unit length of levee flowing up through crack in the top semipervious blanket at berm toe

Q_{LS} Seepage per unit length of levee in the pervious foundation beyond the berm toe

t Maximum berm thickness at landside levee toe

t_x Berm thickness at point x

x Horizontal distance riverward of landside berm toe; may also be a mathematical variable (see Appendix B, Supplement No. 1)

$\bar{X}$ Effective length of riverside top blanket

y	Piezometric head above top of seepage berm
Z_b	Thickness of top blanket
α	A constant (see Equation 63)
γ_b'	Buoyant unit weight of top blanket
γ_t'	Buoyant unit weight of berm
γ_w	Unit weight of water
Δ	A constant (see Equation 64)
ε	Equal to $1/\pi \arctan 1/\sqrt{K}$
θ	A constant (see Equations 36, 52, and 72)
ξ	A constant (see Equation 33a); see Equation 90
ϕ	A variable (see Equation 73)
ψ	A constant (see Equation 92)
ω_B	A constant (see Equation 98)
ω_o	A constant (see Equation 100)
ω_x	A variable (see Equation 95)